Nonlinear evolution of torsional Alfven waves in magnetic flux tubes

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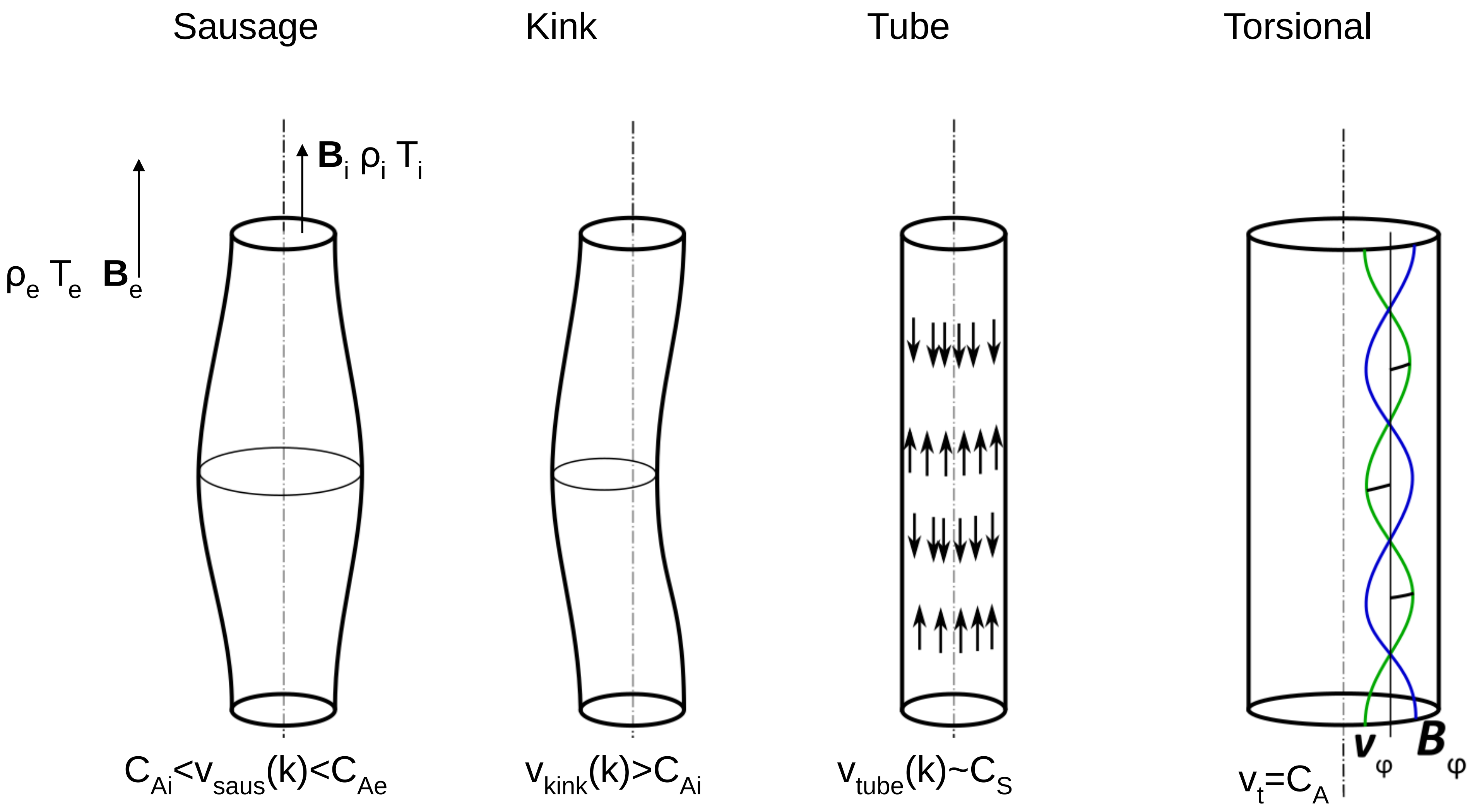
Talk overview

- Introduction;
- Plane waves in 1D: parallel velocities, wave steepening;
- Torsional waves:
  - Numerical setup ;
  - Results of numerical 3D simulations;
- Conclusions

Oscillations in magnetic tubes

0-order thin flux-tube approximation
(Roberts & Webb 1978)



Torsional Alfven waves

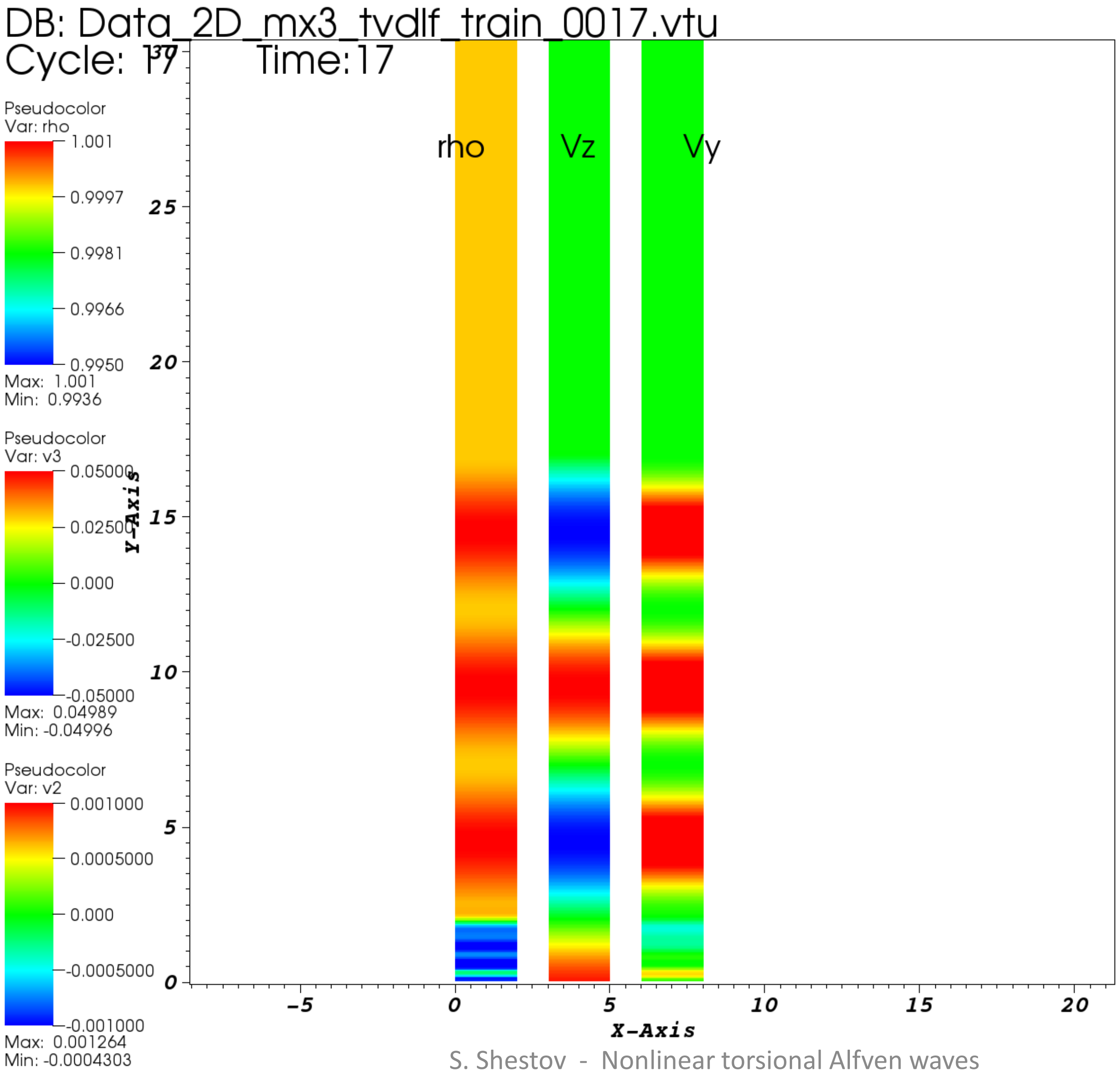
Applications

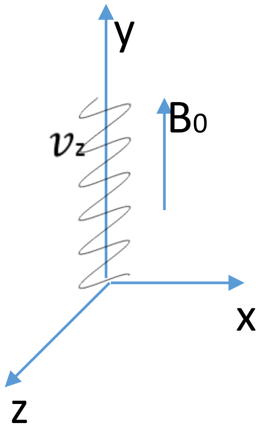
- Freely propagate from lower layers to corona (*Ruderman* 1999; *Copil et al*. 2008);
- Heat coronal plasma treads (*Copil et al*. 2008);
- Large-scale torsional Alfven waves can accelerate electrons (Fletcher & Hudson 2008);
- Accelerate solar wind (Matsumoto & Suzuki 2012);

Studies

- *Vasheghani Farahani et al*. 2011, 2012 – perturb density, wave profile steepening;
- *Fedun et al*. 2011 – magnetic tubes can act as a frequency filter for torsional waves;
- *Murawski et al*. 2015 – efficiency of propagation of torsional Alfven waves in expanding magnetic tubes;

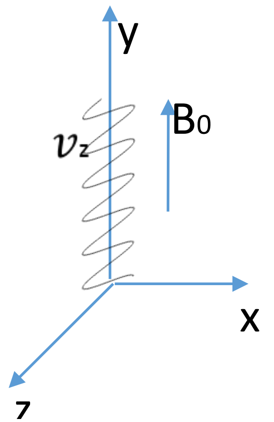
1D plane waves - nonlinear induction $\boldsymbol{v}_y$ ($\boldsymbol{v}_{||}$)



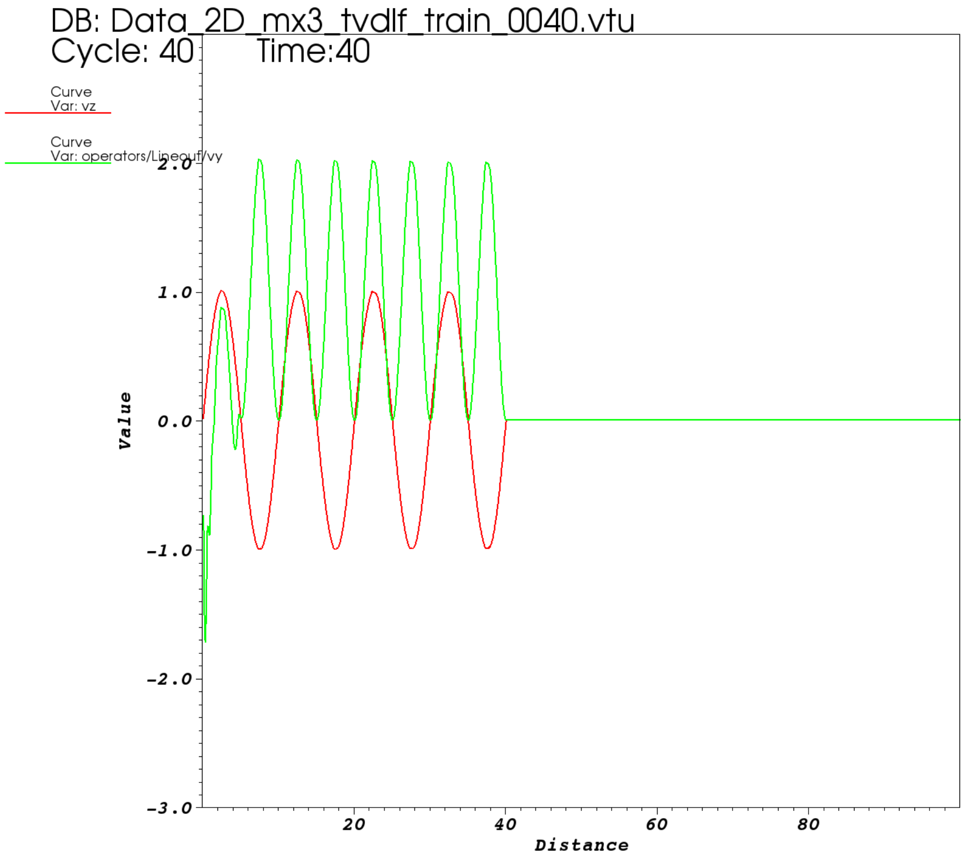


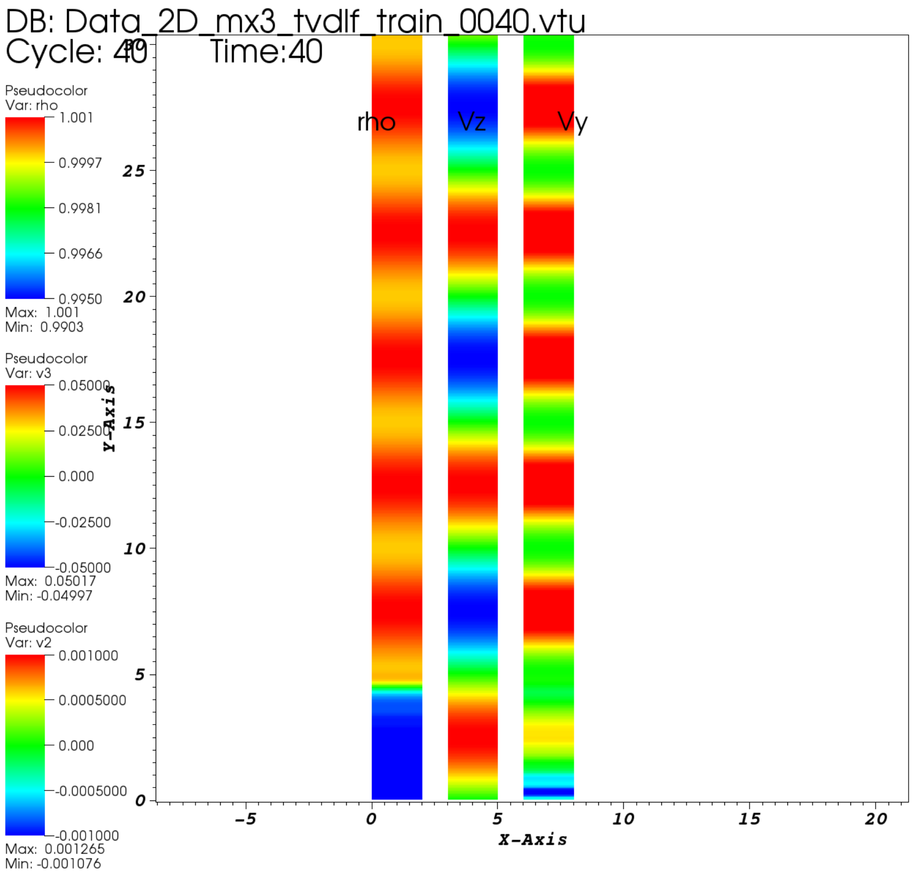
See e.g.
McLaughlin et. al 2011;
Zheng et al. 2016;

1D plane waves - nonlinear induction of $\boldsymbol{v}_y$ ($\boldsymbol{v}_{||}$)

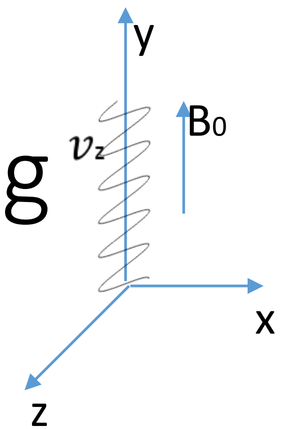


$$v_y = \frac{A^2 C_A}{4(C_A^2 - C_S^2)}\left(1 - \cos\left[2\omega(t - y/C_A)\right]\right)$$

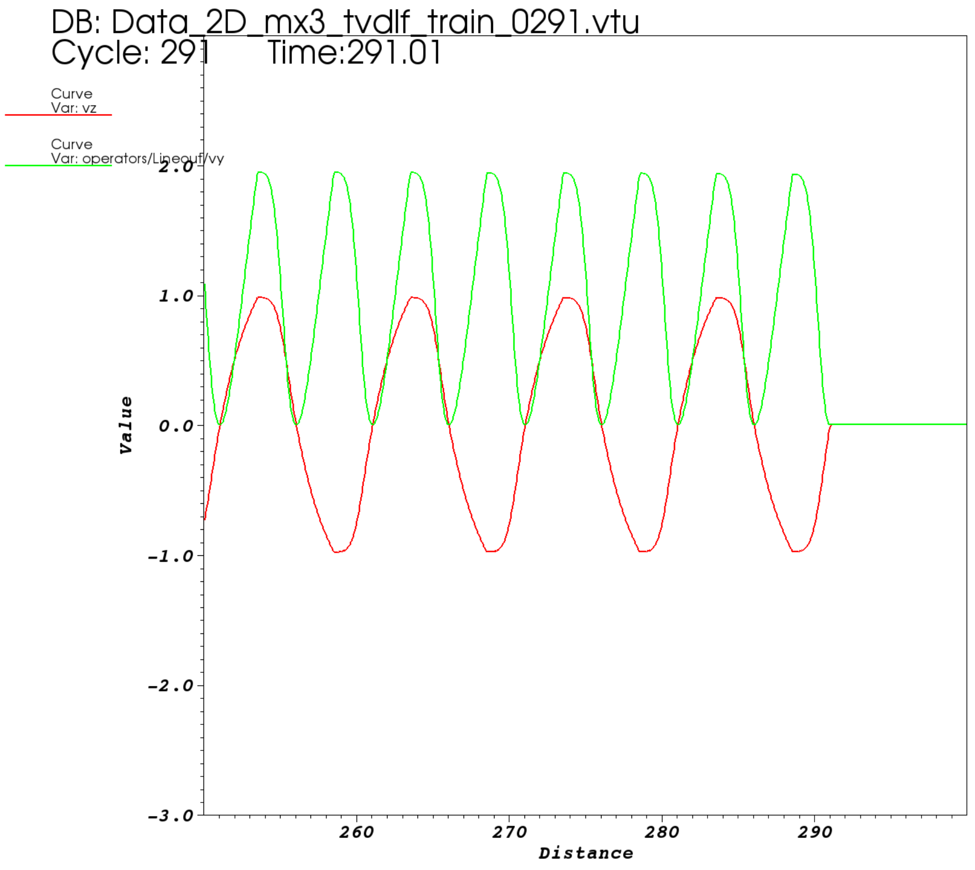


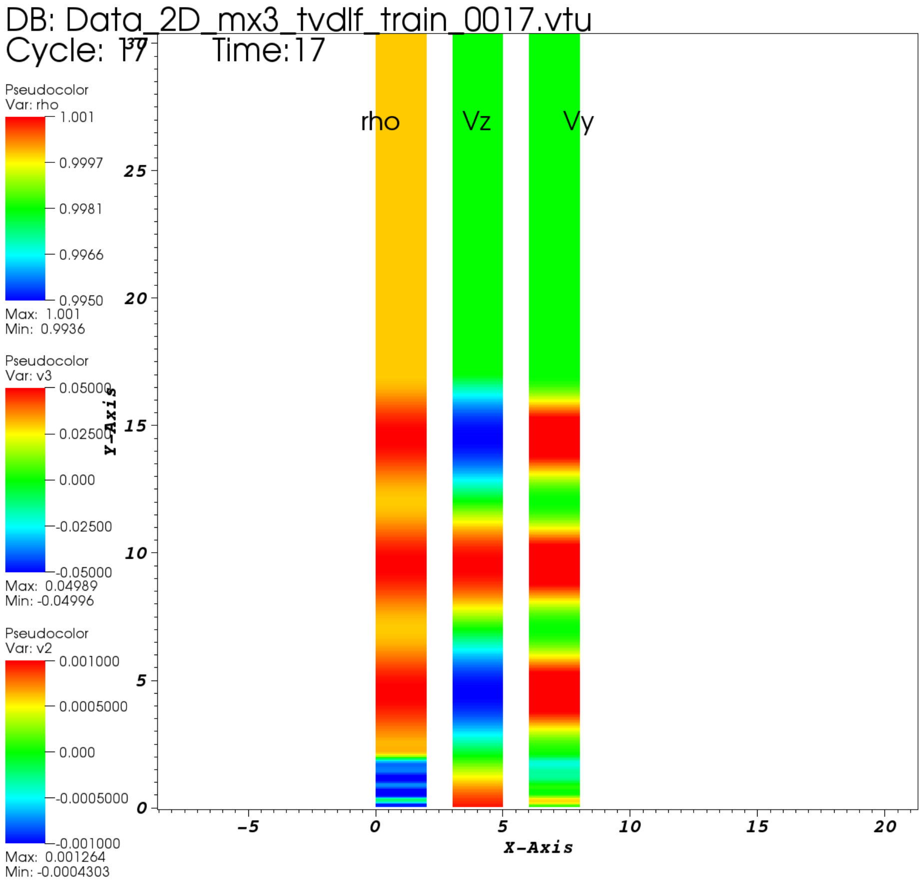


1D plane waves – Cohen-Kulsrud wave steepening



Very late t:





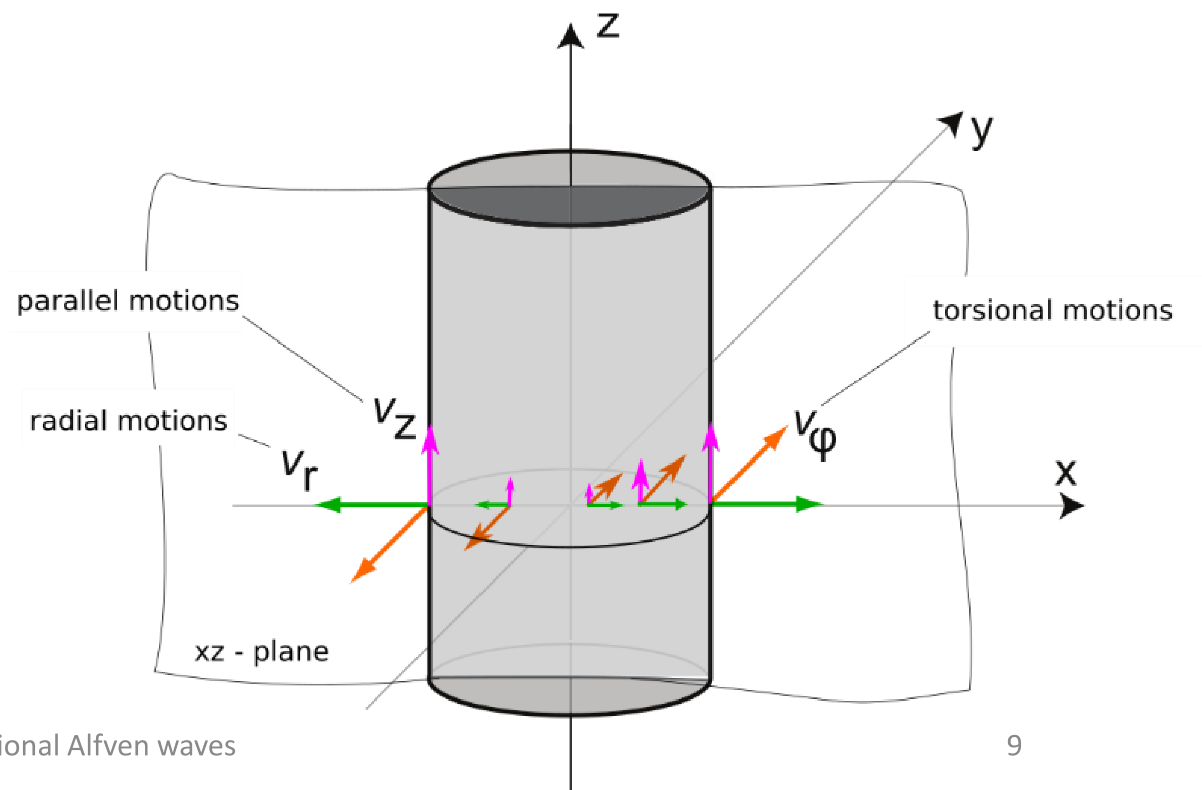
Torsional waves:

- Plane waves <-> wide wavefront (H » λ);
- Torsional waves exist in loops, open field lines (propagating, standing);
- Inapplicability of 0-order thin flux-tube
  → 2nd order flux-tube approximation — (Zhugzhda 1996);
- Consideration in cylindrical reference frame

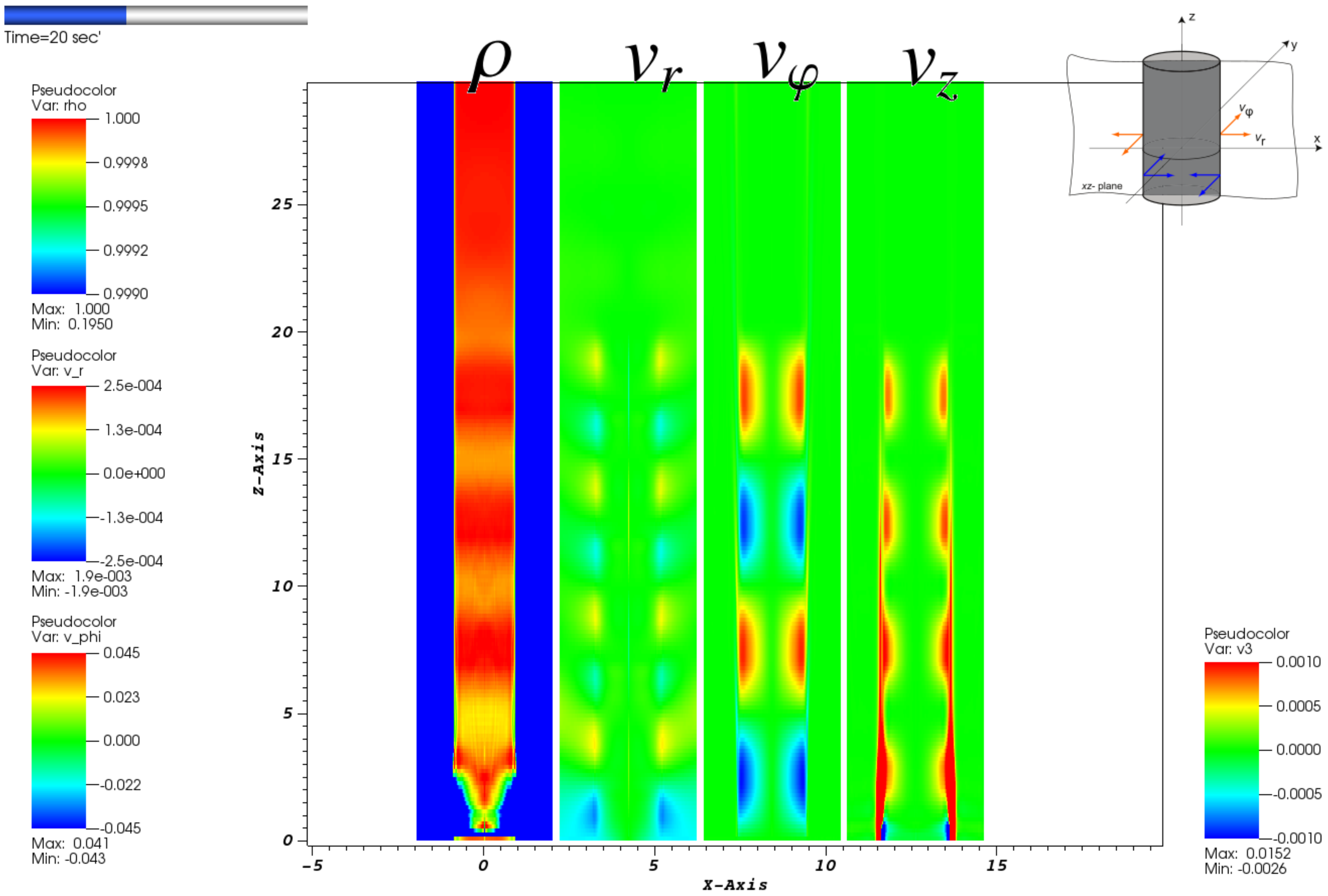
$$\begin{aligned} &\rho \approx \tilde{\rho},\ p \approx \tilde{p} + p_2 r^2,\ v_r \approx V r,\ v_\varphi \approx \Omega r,\ v_z \approx u \\ &B_r \approx B_{r1} r,\ B_\varphi \approx J r,\ B_z \approx \widetilde{B_z} \end{aligned} \qquad (1)$$

Torsional waves – numerical setup

- MPI-AMRVAC code, cylindrical RF, 3D or 2D;
- Straight magnetic flux tube $\rho_e < \rho_i$, R_0 – radius; coronal parameters;
- Perturb B_φ, v_φ as $\sim\omega r$ at bottom boundary;



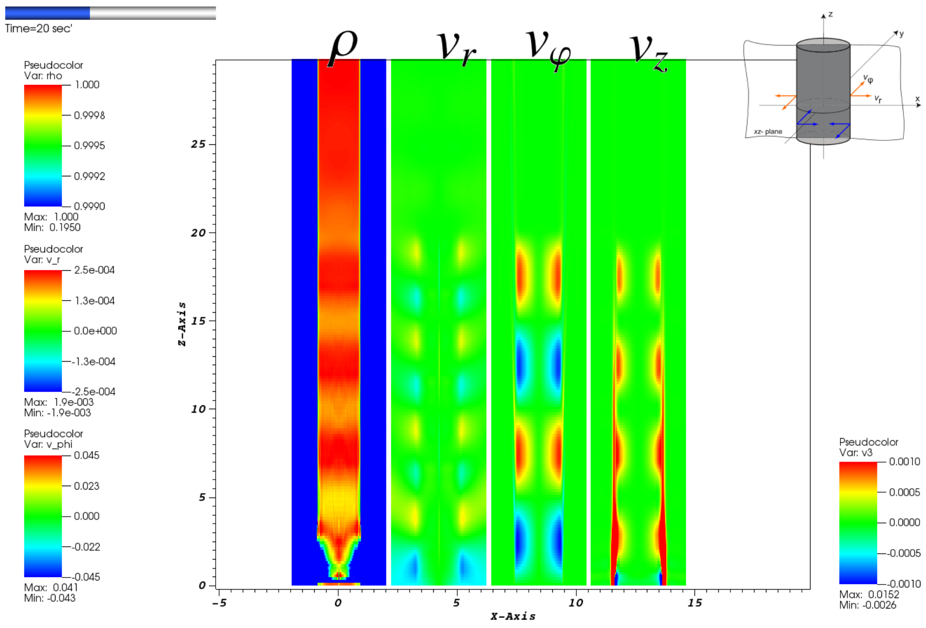
Torsional waves – main characteristics



Torsional waves – radial profiles

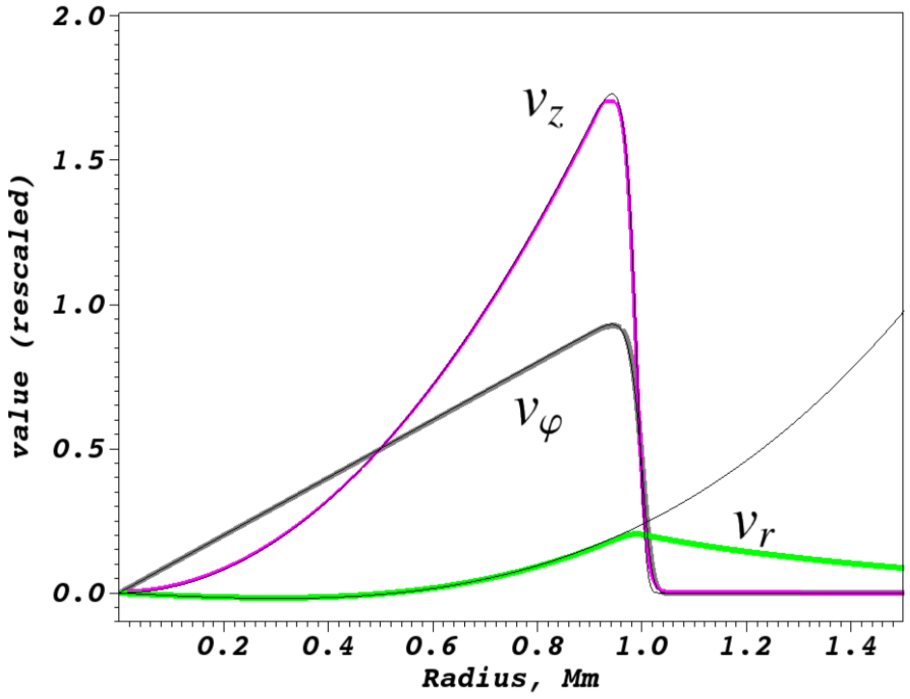
simulated:

$$v_z(r,z,t) = \frac{\Omega_M^2 r^2}{4C_A} \frac{(1-\cos[2\omega(t-z/C_A)])}{\cosh^2\left[(r/R_0)^\alpha\right]}$$



expected:

$$u_p(z,t) = \frac{\Omega_M^2 R^2}{4C_A} \frac{(1-\cos[2\omega(t-z/C_A)])}{\cosh^2\left[(r/R_0)^\alpha\right]}$$

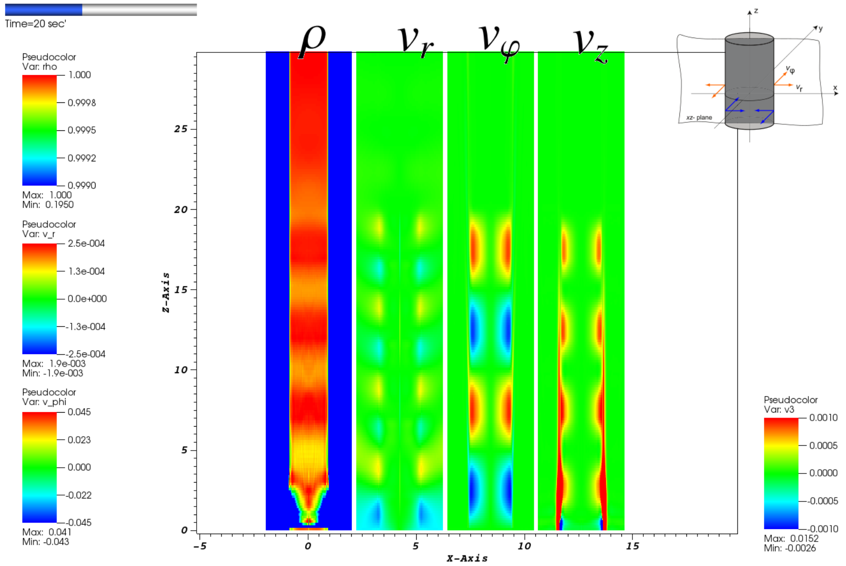


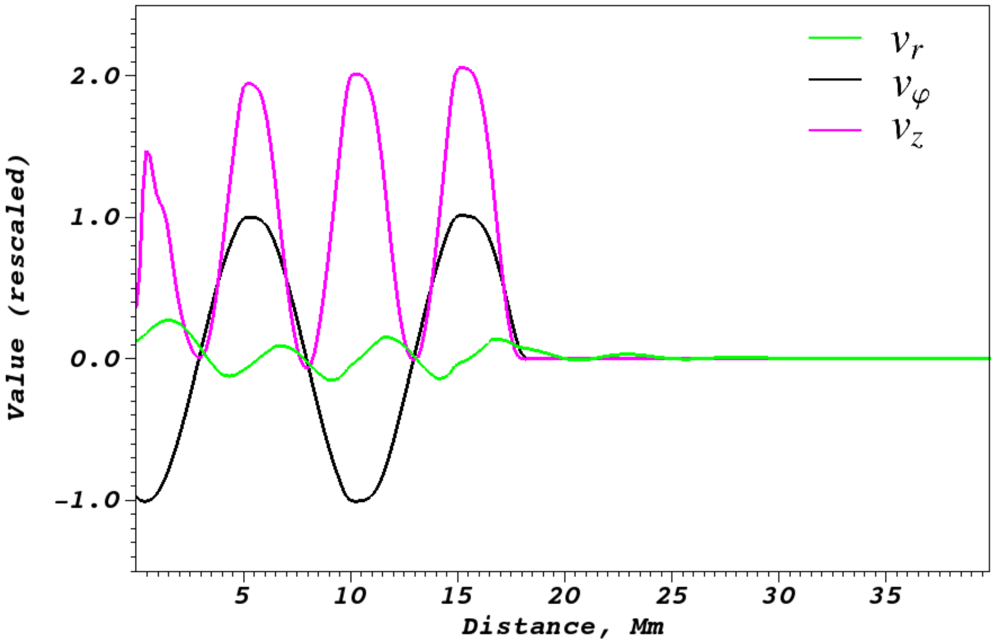
rescaling:

v_φ : $\Omega_M = 0.05$

v_r, v_z : $\frac{\Omega_M^2}{4} = 6.4 \cdot 10^{-4}$

Torsional waves – parallel profiles





Torsional: $$v_z = \frac{\Omega_M^2 r^2}{4C_A}\left(1-\cos\left[2\omega(t-z/C_A)\right]\right)$$

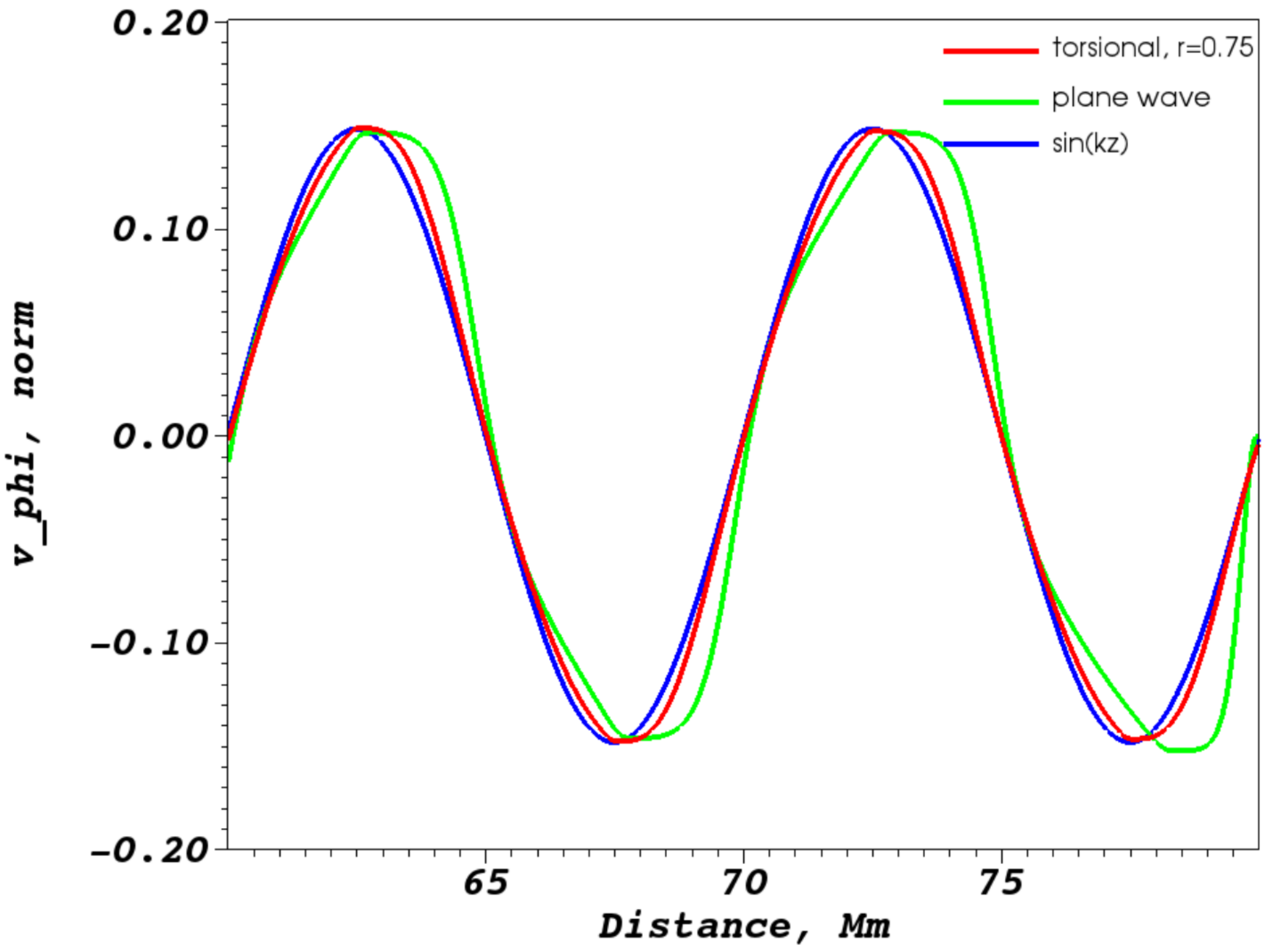
Plane wave: $$u_p = \frac{A^2 C_A}{4(C_A^2 - C_S^2)}\left(1-\cos\left[2\omega(t-y/C_A)\right]\right)$$

rescaling:

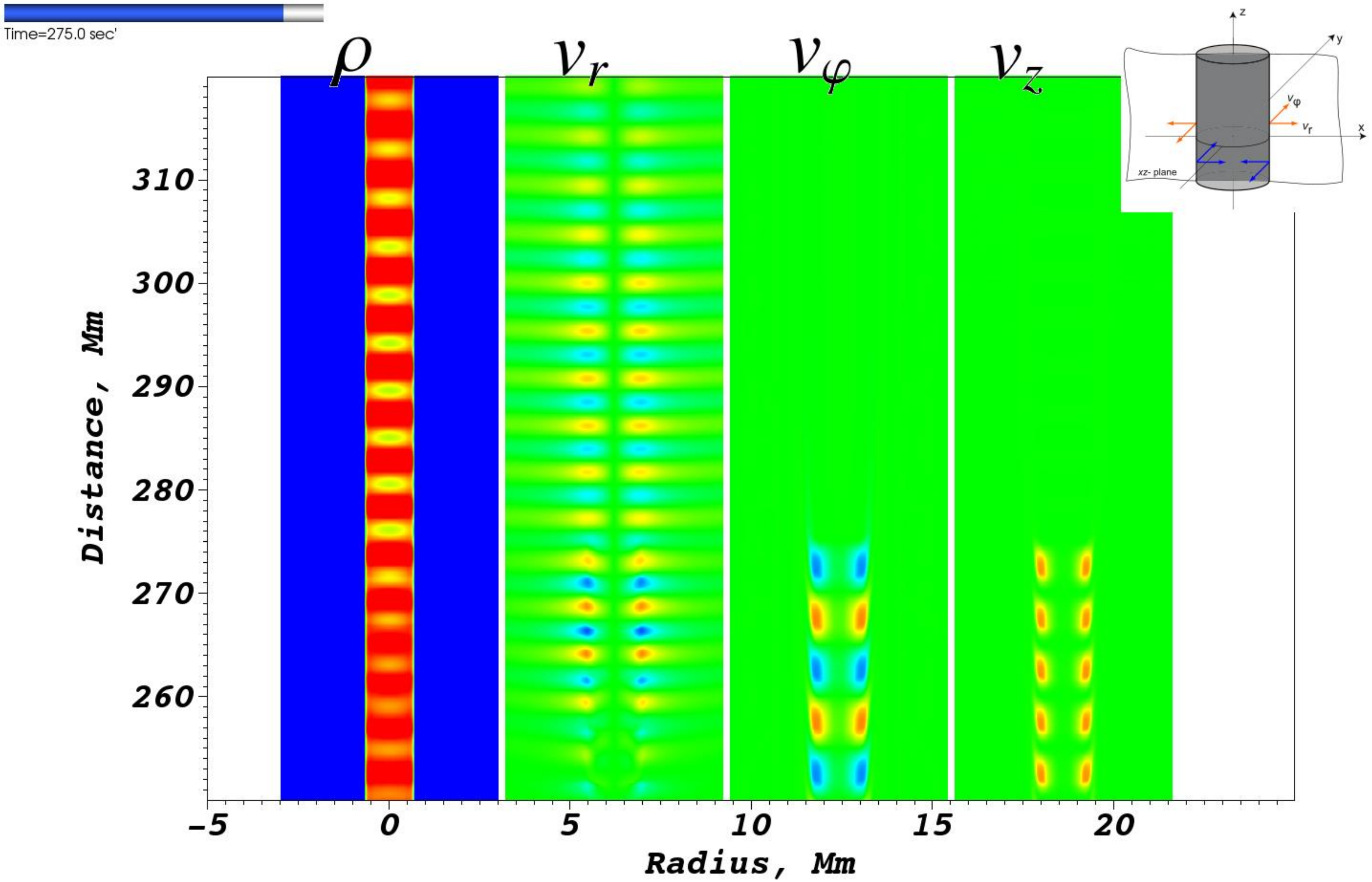
v_φ : $\Omega_M = 0.05$

v_r, v_z : $\frac{\Omega_M^2}{4} = 6.4\cdot 10^{-4}$

Torsional waves – wave profile steepening

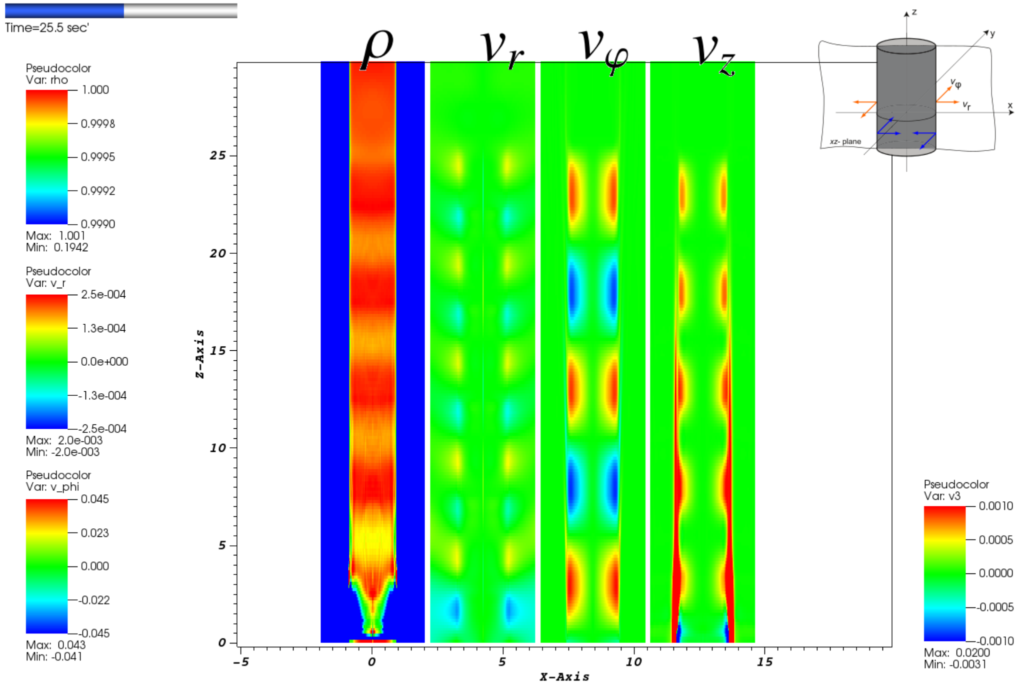


Torsional waves – generation of sausage waves



Summary

- Torsional wave in general conform to theory
- Nonlinear effects -> v_r, v_z, perturb p, ρ;
- The differences with the case of plane waves:
  - $v_z \sim r^2$ — importance of the tube`s finite radius;
  - efficiency does not depend on T, plasma β;
  - wave steepening (weaker then for plane);
  - development of FMA wave;



see *Shestov et al*. ApJ 2017

Appendix

1D plane waves - derivation

1) MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$
$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \left(\frac{\nabla \times \boldsymbol{B}}{\mu}\right) \times \boldsymbol{B} + \nu \nabla \cdot \mathbf{S},$$
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B},$$
$$\frac{\partial p}{\partial t} + (\boldsymbol{v}\cdot\nabla) p = -\gamma p \nabla \cdot \boldsymbol{v} + \frac{\gamma - 1}{\sigma} |j|^2 + \nu(\gamma - 1) Q_{\mathrm{visc}}, \quad (1)$$

2) Cartesian coordinates, small perturbations

$$\rho_0 = \rho_0(x)\ ,\ p_0 = \text{constant}\ ,\ \boldsymbol{B}_0 = (0, B_0, 0)\ ,\ \boldsymbol{v}_0 = \boldsymbol{0}$$

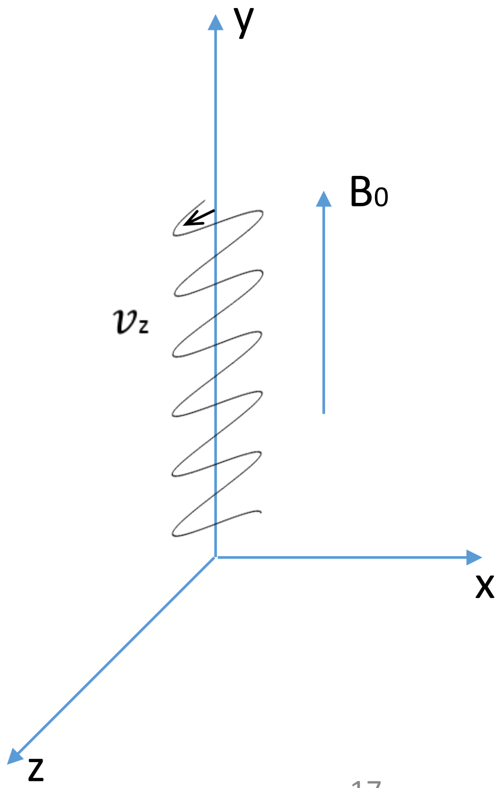
$\rho \to \rho_0 + \rho$

$v \to v_0 + v$

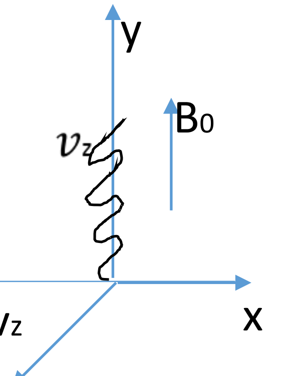
$B \to B_0 + B$

Linearised equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho_0 v_x) + \rho_0 \frac{\partial v_y}{\partial y} = N_1,$$
$$\rho_0 \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} + \frac{B_0}{\mu}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) + V_1 = N_2,$$
$$\rho_0 \frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} + V_2 = N_3,$$
$$\rho_0 \frac{\partial v_z}{\partial t} - \frac{B_0}{\mu}\frac{\partial B_z}{\partial y} + V_3 = N_4\ ,$$
$$\frac{\partial B_x}{\partial t} - B_0 \frac{\partial v_x}{\partial y} - R_1 = N_5,$$
$$\frac{\partial B_y}{\partial t} + B_0 \frac{\partial v_x}{\partial x} - R_2 = N_6,$$
$$\frac{\partial B_z}{\partial t} - B_0 \frac{\partial v_z}{\partial y} - R_3 = N_7,$$
$$\frac{\partial p}{\partial t} + \gamma p_0 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = N_8,$$



1D plane waves - derivation



Linearised equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho_0 v_x) + \rho_0 \frac{\partial v_y}{\partial y} = N_1,$$

$$\rho_0 \frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} + \frac{B_0}{\mu}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) + V_1 = N_2,$$

$$\rho_0 \frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} + V_2 = N_3,$$

$$\rho_0 \frac{\partial v_z}{\partial t} - \frac{B_0}{\mu}\frac{\partial B_z}{\partial y} + V_3 = N_4\ ,$$

$$\frac{\partial B_x}{\partial t} - B_0 \frac{\partial v_x}{\partial y} - R_1 = N_5,$$

$$\frac{\partial B_y}{\partial t} + B_0 \frac{\partial v_x}{\partial x} - R_2 = N_6,$$

$$\frac{\partial B_z}{\partial t} - B_0 \frac{\partial v_z}{\partial y} - R_3 = N_7,$$

$$\frac{\partial p}{\partial t} + \gamma p_0 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = N_8,$$

Right-hand side – nonlinear terms

$$N_1 = -\frac{\partial}{\partial x}(\rho v_x) - \frac{\partial}{\partial y}(\rho v_y),$$

$$N_2 = -\frac{B_z}{\mu}\frac{\partial B_z}{\partial x} - \rho \frac{\partial v_x}{\partial t} - (\rho_0 + \rho)\left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y}\right) v_x - \frac{B_y}{\mu}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right),$$

$$N_3 = -\frac{B_z}{\mu}\frac{\partial B_z}{\partial y} - \rho \frac{\partial v_y}{\partial t} - (\rho_0 + \rho)\left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y}\right) v_y + \frac{B_x}{\mu}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right),$$

$$N_4 = -\rho \frac{\partial v_z}{\partial t} - (\rho_0 + \rho)\left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y}\right) v_z + \frac{B_y}{\mu}\frac{\partial B_z}{\partial y} + \frac{B_x}{\mu}\frac{\partial B_z}{\partial x},$$

$$N_5 = \frac{\partial}{\partial y}\left(v_x B_y - v_y B_x\right),$$

$$N_6 = -\frac{\partial}{\partial x}\left(v_x B_y - v_y B_x\right),$$

$$N_7 = \frac{\partial}{\partial x}(v_z B_x - v_x B_z) + \frac{\partial}{\partial y}\left(v_z B_y - v_y B_z\right),$$

Wave equations for v_x, v_y, v_z

$$\left[\frac{\partial^2}{\partial t^2} - \left(v_A^2 + c_S^2\right)\frac{\partial^2}{\partial x^2} - v_A^2 \frac{\partial^2}{\partial y^2}\right] v_x - c_s^2 \frac{\partial^2 v_y}{\partial x \partial y}$$
$$= \frac{1}{\rho_0}\left[-\frac{\partial V_1}{\partial t} - \frac{\partial N_8}{\partial x} + \frac{\partial N_2}{\partial t} - \frac{B_0}{\mu}\left(\frac{\partial N_6}{\partial x} + \frac{\partial R_2}{\partial x} - \frac{\partial N_5}{\partial y} - \frac{\partial R_1}{\partial y}\right)\right],$$

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial y^2}\right) v_y - c_s^2 \frac{\partial^2 v_x}{\partial x \partial y}$$
$$= \frac{1}{\rho_0}\left(-\frac{\partial V_2}{\partial t} + \frac{\partial N_3}{\partial t} - \frac{\partial N_8}{\partial y}\right),$$

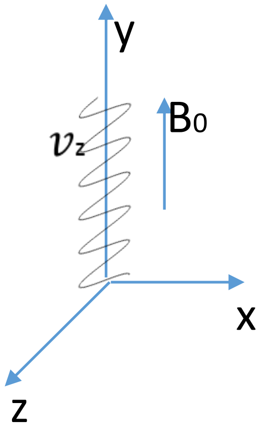
$$\left(\frac{\partial^2}{\partial t^2} - v_A^2 \frac{\partial^2}{\partial y^2}\right) v_z$$
$$= \frac{1}{\rho_0}\left[-\frac{\partial V_3}{\partial t} + \frac{\partial N_4}{\partial t} + \frac{B_0}{\mu}\left(\frac{\partial N_7}{\partial y} + \frac{\partial R_3}{\partial y}\right)\right],$$

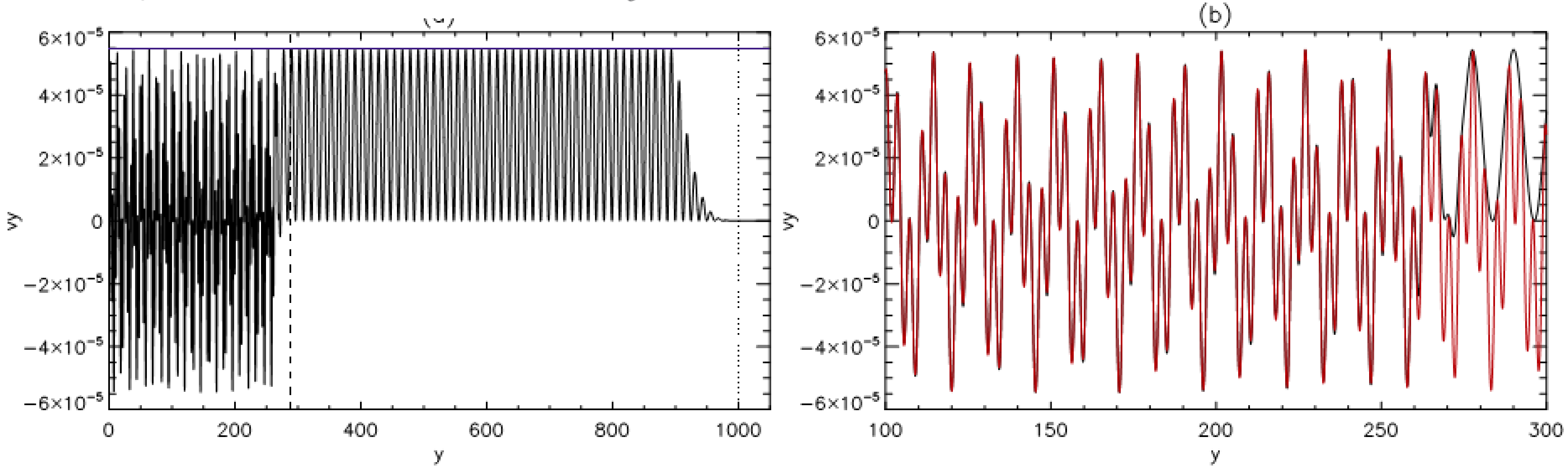
1D plane waves - nonlinear induction $\boldsymbol{v}_y$ ($\boldsymbol{v}_{||}$)

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial y^2}\right) v_y = \frac{1}{\rho_0}\frac{\partial N_3}{\partial t} = -\frac{1}{\mu\rho_0}\frac{\partial^2}{\partial t \partial y}\left(\frac{B_z^2}{2}\right).$$

Thus, the analytical solution for v_y is:

$$v_y = \begin{cases} \frac{A^2 v_A}{4(v_A^2 - c_s^2)} \left(\cos[2\omega(t - y/c_s)] - \cos[2\omega(t - y/v_A)]\right) & 0 < y < c_s t\,, \\ \frac{A^2 v_A}{4(v_A^2 - c_s^2)} \left(1 - \cos[2\omega(t - y/v_A)]\right) & c_s t < y < v_A t, \\ 0 & y > v_A t. \end{cases}$$





See e.g.
McLaughlin et. al 2011;
Zheng et al. 2016;

Torsional waves: MHD equations in cylindrical RF

$$\rho\left(\frac{\partial V}{\partial t}+u\frac{\partial V}{\partial z}+V^2-\Omega^2\right)+2p_2+\frac{1}{2\pi}\left(J^2+B_zB_{z2}\right)-$$
$$-\frac{1}{4\pi}B_z\frac{\partial B_{r1}}{\partial z}=0 \quad (2)$$

$$\frac{\partial\Omega}{\partial t}+u\frac{\partial\Omega}{\partial z}+2V\Omega+\frac{J}{4\pi\rho}\frac{\partial B_z}{\partial z}-\frac{B_z}{4\pi\rho}\frac{\partial J}{\partial z}=0 \quad (3)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}\right)+\frac{dp}{dz}=0 \quad (4)$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial z}+2\rho V=0 \quad (5)$$

$$\frac{\partial B_{r1}}{\partial t}+\frac{\partial(uB_{r1})}{\partial z}-\frac{\partial(VB_z)}{\partial z}=0 \quad (6)$$

$$\frac{\partial J}{\partial t}+\frac{\partial(uJ)}{\partial z}-\frac{\partial\Omega B_z}{\partial z}+2VJ-2\Omega B_{r1}=0 \quad (7)$$

$$\frac{\partial B_z}{\partial t}+u\frac{\partial B_z}{\partial t}+2B_zV=0 \quad (8)$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)\frac{p}{\rho^\gamma}=0 \quad (9)$$

$$2B_{r1}+\frac{\partial B_z}{\partial z}=0 \quad (10)$$

$$B_zA=const$$

$$p+\frac{B_z^2}{8\pi}-\frac{A}{2\pi}\left[\rho\left(\frac{\partial V}{\partial t}+u\frac{\partial V}{\partial z}+V^2-\Omega^2\right)+\right.$$
$$\left.+\frac{1}{4\pi}\left(J^2-\frac{1}{4}\left(\frac{\partial B_z}{\partial z}\right)^2+\frac{B_z}{2}\frac{\partial^2 B_z}{\partial z^2}\right)\right]=p_T^{ext},$$

Torsional waves – nonlinear induction of v_z ($v_{||}$)

Linearization: $\rho_0,\ p_0,\ B_{z0},\ A_0$

$$u_0 = V_0 = J_0 = \Omega_0 = B^0_{r1} = 0.$$

Linear regime:

Nonlinear regime:

Rotational variables

$$\frac{\partial \Omega}{\partial t} - \frac{B_{z0}}{4\pi\rho_0}\frac{\partial J}{\partial z} = 0$$

$$\frac{\partial J}{\partial t} - B_{z0}\frac{\partial \Omega}{\partial z} = 0$$

$$\left[\frac{\partial^2}{\partial t^2} - C_A^2\frac{\partial^2}{\partial z^2}\right] J = 0,$$

$$J = J_M \cos(\omega t - kz)$$
$$\Omega = \Omega_M \cos(\omega t - kz),$$

$$B_\varphi = J_M r \cos(\omega t - kz)$$
$$v_\varphi = \Omega_M r \cos(\omega t - kz)$$

Compressible variables

$$\rho_0\frac{\partial u}{\partial t} + \frac{\partial p}{\partial z} = 0$$

$$\frac{\partial \rho}{\partial t} + \rho_0\frac{\partial u}{\partial z} + 2\rho_0 V = 0$$

$$\frac{\partial B_z}{\partial t} + 2B_{z0}V = 0$$

$$\frac{\partial p}{\partial t} - C_S^2\frac{\partial \rho}{\partial t} = 0$$

$$\frac{1}{A_0}\frac{\partial A}{\partial t} = -\frac{1}{B_{z0}}\frac{\partial B_z}{\partial t}$$

$$p + \frac{B_{z0}}{4\pi}B_z = p_{ext}$$

$$\frac{1}{C_S^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial z^2} + \frac{1}{C_A^2}\frac{\partial^2 u}{\partial t^2} = 0$$

$$\rho_0\frac{\partial u}{\partial t} + \frac{\partial p}{\partial z} = -\frac{R^2}{4\pi}J\frac{\partial J}{\partial z}$$

$$\frac{\partial^2 u}{\partial t^2} - C_T^2\frac{\partial^2 u}{\partial z^2} = -\frac{R^2 C_T^2}{4\pi\rho_0 C_S^2}\frac{\partial}{\partial t}\left(J\frac{\partial J}{\partial z}\right)$$

Torsional waves – nonlinear induction of $\boldsymbol{v}_z$ ($\boldsymbol{v}_{||}$)

Linear regime:

$$\frac{\partial^2 u}{\partial t^2} - C_T^2 \frac{\partial^2 u}{\partial z^2} = 0$$

Nonlinear regime:

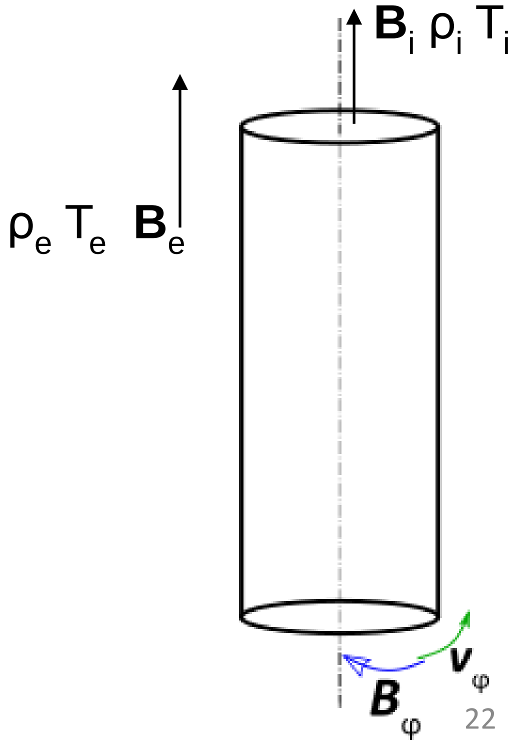
$$\frac{\partial^2 u}{\partial t^2} - C_T^2 \frac{\partial^2 u}{\partial z^2} = \frac{R^2 C_T^2}{4\pi\rho_0 C_S^2} \frac{\partial}{\partial t}\left(J \frac{\partial J}{\partial z} \right)$$

Applying torsional perturbation (z=0), we obtain a solution:

$$u = \begin{cases} u_T + u_p, & 0 < z < C_T t, \\ u_p, & C_T t < z < C_A t, \\ 0, & C_A t < z, \end{cases}$$

$$u_p = \frac{R^2 \Omega_M^2}{4 C_A} \left(1 - \cos\left[2\omega(t - z/C_A)\right]\right),$$ Ponderomotive wave

$$u_T = \frac{R^2 \Omega_M^2}{4 C_A} \left(\cos\left[2\omega(t - z/C_T)\right] - 1\right),$$ Tube wave



Numerical setup

Normalized values:

B_0 = 20G;
L = 1 Mm;
ρ = 1.67e-15 g/cm^3;
n_e = 1e9 cm^{-3};
v_0 = 1.380 km/sec;
t_0 = 0.724 sec;

Grid sizes:

128 x 64 x 256 in r,phi,z;
r – 0-2 Mm; z – 0-40 Mm;

16384 x 384 in r,z;
r – 0-3 Mm, z – 0-600 Mm;

$$n_e(r) = n_\infty + \frac{(n_0 - n_\infty)}{\cosh^2[(r/R_0)^\alpha]},$$

$$B_z(r) = B_0 \left\{1 + \frac{8\pi k_B T(n_0 - n_\infty)}{B_0^2}\left(1 - \frac{1}{\cosh^2[(r/R_0)^\alpha]}\right)\right\}^{1/2}$$

- equilibrium state

$$B_\varphi(r) = \frac{J_M r \sin(\omega t)}{\cosh^2[(r/R_0)^\alpha]}$$

$$v_\varphi(r) = \frac{\Omega_M r \sin(\omega t)}{\cosh^2[(r/R_0)^\alpha]},$$

- wave driver