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Subphotospheric convection and magnetic activity dependence on metallicity and age: models and tests

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Abstract. We present an extensive study on the dependence of the convective turnover time ($\tau_{\rm conv}$) on the stellar metallicity and age for main-sequence stars of masses 0.6–1.6 $M_\odot$. To this aim we have used and compared predictions by stellar models based on the classical Mixing Length Theory and models incorporating a Full Spectrum of Turbulence treatment of subphotospheric convection. We show that the metallicity effect is relevant for dG stars but negligible for dK stars, while stellar age is important when computing the turnover times for red dwarfs younger than $\log t \sim 8.5$ yr. A scatter by up to a factor 3 could be spuriously introduced in the activity vs. Rossby number relationships if such effects are neglected. We have tested our model predictions on the metallicity effect by comparison with the observed median X-ray luminosities for dG and dK stars in the open clusters NGC 2516, the Pleiades, and Blanco 1, having similar age but likely different metal contents. We show that our model predictions, taking into account also the dependence of the coronal plasma emissivity on the metal abundance, are in agreement with the observations in all cases except for the dK stars in the metal-rich cluster Blanco 1, where our assumption of a rotation period distribution similar to those of the other cluster need to be confirmed by future observations.

Key words. Convection - Stars: abundances - Stars: coronae - X-rays: stars

1. Introduction

Chromospheric and coronal emission in late-type stars is the end result of a long chain of processes by which magnetic fields are generated in the stellar interior and then transported to the stellar photosphere, where they allow non-radiative energy to be deposited in the outer atmosphere and finally radiated away. It is this radiation, observed at optical, UV and X-ray wavelengths, which is used to measure the efficiency of the dynamo mechanism, the primary responsible of the generation and amplification of magnetic fields. In fact, there is yet no consistent theoretical dynamo model which would allow the prediction of the magnetic flux emerging at photospheric level, and hence to estimate what fraction of the total energy budget goes into plasma heating and radiation. For this reason, the dynamo strength is often defined on an empirical basis, as some monotonically increasing function of the outer atmospheric radiative losses (Schrijver & Zwaan 2000).

Observations have shown that chromospheric and coronal activity – and, by inference, a magnetic dynamo – is a characteristic of essentially all rotating stars with a subphotospheric convection zone; moreover, for late-type main-sequence stars, the emission level increases for increasing rotation rate, up to a "saturation" value in which the UV/X-ray luminosity is of the order of 10^{-3} times the stellar bolomentric luminosity (Vilhu 1984, Vilhu & Walter 1987). For non-saturated stars, the dynamo strength then appears to be a function of the stellar rotation rate, but it must depend also on some property of the convective envelope.

One of the first studies which unveiled a quantitative link between magnetic activity in main-sequence stars and the stellar internal structure was carried out by Noyes et al. (1984), using observations of the chromospheric Ca II H and K emission lines from 41 main-sequence stars with known rotation periods. They found a good correlation between the ratio of the Ca II HK to bolometric flux density, $R'_{\rm HK}$ (corrected for a line-wing photospheric contribution), and the rotation period, $P_{\rm rot}$, divided by a color-dependent scaling function, $\tau(B-V)$, approximately equal to the convective turnover time, $\tau_{\rm conv}$, computed by Gilman (1980) from a mixing-length convection model with $\alpha \sim 2$, at one pressure scale height above the bottom of the convection zone. Following this semi-empirical approach, the Rossby number, $R_{\rm O} = P_{\rm rot}/\tau_{\rm conv}$, was suggested as a quantity having a key role in predicting the surface magnetic activity in low main-sequence stars.

From a theoretical point of view, the above finding is in agreement with a rough prediction of linear kinematic dynamo models, based on dimensional arguments. The existence of dynamo modes with positive growth rates for the mean magnetic field, depends on the value of a "dynamo number", which is essentially the ratio between magnetic field generation and diffusion terms (Parker 1979): $N_{\rm D} = \alpha(\nabla_r\Omega)d^4/\eta^2$, being α the product of the mean elicity with the convective turnover time, $\nabla_r\Omega$ the radial gradient of the angular velocity, d the characteristic scale lenght of convection, and η the turbulent magnetic diffusivity. Assuming a linear scaling of $\nabla_r\Omega$ with the surface rotation rate, $N_{\rm D}$ becomes proportional to ${R_{\rm O}}^{-2}$ (Durney & Latour 1978), thus explaining – at least, qualitatively – the observed correlation of $R_{\rm O}$ with the magnetic activity level.

Following the work of Noyes et al. (1984), several studies have tested the activity vs. Rossby number relationship using different stellar samples and activity indicators. In particular, relationships between the coronal X-ray emission level and the Rossby number have been investigated by Micela et al. (1984), Schmitt et al. (1985), and Maggio et al. (1987) for selected samples of F and G-type dwarfs, and by Dobson & Radick (1989), Stepien (1994), Hempelmann et al. (1995), and Randich (1999) – to cite a few – for heterogenous samples of F to M stars, including members of open clusters.

Up to date, there is yet no general consensus on what is the best coronal activity index to use (X-ray luminosity, surface flux, or X-ray to bolometric flux ratio), and hence no common agreement on the functional form of the correlation of this index with the Rossby number. Nonetheless, $R_{\rm O}$ is often considered as the dynamo-related parameter best suited as predictor of the coronal emission level. To this aim, $R_{\rm O}$ is usually computed by dividing an *observed* $P_{\rm rot}$ to an *empirical* $\tau_{\rm conv}$, evaluated as a function of the $B-V$ color with the Noyes et al. (1984) formula, or taken from the work of Stepien (1994). In this respect, a critical issue is the applicability of such empirical $\tau_{\rm conv}$ to stellar samples with characteristics very different from those on which the original $\tau_{\rm conv}$ values were based, and in particular to samples of very young stars, or stars in open clusters with known non-solar metallicity.

On the other hand, the few available *theoretical* computations of the convective turnover time (Gilman 1980; Rucinsky & Vandenberg 1986, 1990; Gilliland 1985; Kim & Demarque 1996), are all based on the Mixing-Lenght Theory (MLT) of convection, and they depend rather sensitively on the ill-defined parameter α, i.e. the ratio of mixing length to pressure scale height. So, the first issue we have addressed in the present work is how the $\tau_{\rm conv}$ based on the classical mixing-lenght theory compares with new theoretical computations based on the more physically sound Full Spectrum of Turbulence (FST) theory of convection by Canuto et al. (1996).

The second step was to explore the sensitivity of the theoretical $\tau_{\rm conv}$ to stellar metallicity or age, two crucial parameters in determining the characteristics of the sub-photospheric convection zone in late-type stars. As a final step, we have drawn model predictions on the dependence of the Rossby number on the above parameters, for stars of different mass or color, and hence on the observability of such effects by means of stellar X-ray studies.

In Sect. 2 we present the stellar structure models used for the present work, and discuss some technical issues related to the computation of the convective turnover time; then, in Sect. 3, we compare our theoretical $\tau_{\rm conv}$ with the empirical values vs. $B-V$ computed by Noyes et al. (1984) and by Stepien (1994); Sect. 4 and Sect. 5 are devoted to explore the metallicity and age effects, respectively, on $\tau_{\rm conv}$ and hence on the Rossby number, while in Sect. 6 we propose some tests to check our model predictions, and we present preliminary results of the comparison between model predictions and observational X-ray data; finally, Sect. 7 is devoted to a summary and conclusions.

2. The models

For our study, we have employed stellar structure models constructed by means of the ATON2.0 code, whose main physical inputs are described in Ventura et al. (1998). Here we recall the way in which convection is approached, which is the key to understand the results obtained for the convective turnover time. The models cover the mass range $0.6 - 1.6\,{\rm M}_\odot$, in steps of 0.1 $\rm M_\odot$, and the metallicities, $Z = 0.01$, 0.017 (solar value) and 0.03 have been considered. Each model has been evolved from the pre-main-sequence phase up to an age of $\sim 10^9$ yr, but for the present purpose we have studied only the characteristics of convection at two selected ages, $\log t = 7.3$ yr, corresponding to a ZAMS star of mass $1.3{\rm M}_\odot$, and $\log t = 8.5$ yr, which is approximately the time of exit of a $1.6\,M_\odot$ star from the main-sequence.

We employ either the Mixing Length Theory (MLT, Böhm-Vitense 1958) or the Full Spectrum of Turbulence (FST) treatment, in which the fluxes are computed following Canuto et al. (1996, CGM). A small overshooting distance of $0.1H_{\rm p}$ is added in all models. One important characteristic of the FST model is that the scale of convection is the distance z between the layer in exam and the top of convection. Overall, the FST model convection is much more efficient than in the classical MLT case, and hence it requires a lower superadiabaticity level to reach the same effect.

Following Kim & Demarque (1996), we have considered two ways of evaluating the convective turnover time. The first way yields a "global" convective turnover time, defined as:

$$\tau_{\rm glob} = \int_{R_{base}}^{R_{\rm top}} \frac{{\rm d}r}{v_{\rm conv}} \tag{1}$$

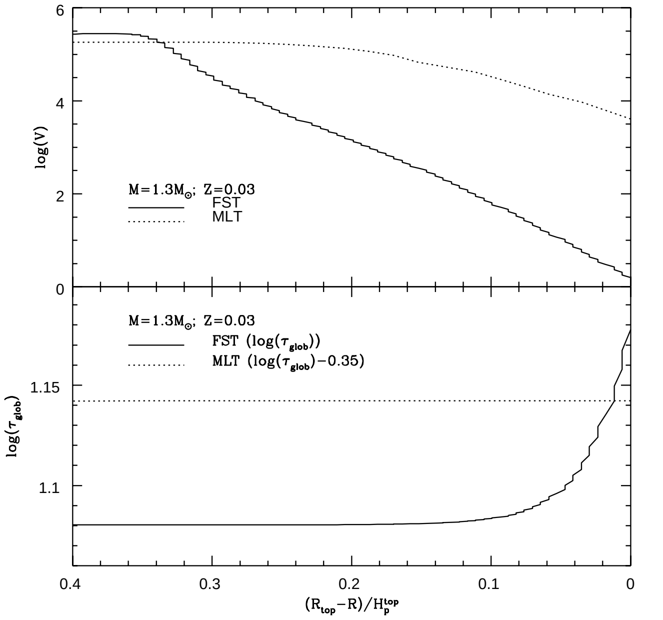

Fig. 1. We show in the top panel the logarithm of the velocity (in cm s^{-1}), and in the bottom panel the logarithm of $\tau_{\rm glob}$ (in days), for an MLT and an FST model of $1.3{\rm M}_\odot$, as a function of the distance from the top of the convection zone (in units of the pressure scale height, $H_{\rm p}$). In the most external layers, the FST model convection is much less efficient than in the MLT case, so the velocity becomes much smaller and $\tau_{\rm glob}$ increases by ~ 0.1 dex. Note that, in the MLT case, $\tau_{\rm glob}$ has been shifted by 0.35 dex for plotting purposes.

where R_0 and $R_{\rm top}$ denote, respectively, the base and the top of the convective region. Since the FST model describes the whole spectrum of energy of the eddies, the velocity $v_{\rm conv}$ is evaluated as the average of the convective velocities of the eddies, computed according to Eq. 80 and 81 in CGM. In the MLT case, the velocity is the speed of the unique convective eddy present in the treatment, and it is computed according to Böhm-Vitense (1958). The latter is a relatively large eddy ($\sim H_{\rm p}$ in size), and the corresponding one in the FST model is also relatively slow: because of the intrinsically different way in which convective velocities are evaluated in the two models, the convective turnorver times in the FST case turn out to be systematically shorter than in the MLT case.

When dealing with FST models, great care has to be taken in the computation of Eq. 1. This is due to the great sensitivity of $v_{\rm conv}$ to the upper extreme of the integral. Figure 1 shows the velocity in the top layers of a model of $1.3{\rm M}_\odot$. The bottom panel then shows how $\tau_{\rm glob}$ changes by adopting, as extreme of the integral, values closer and closer to $R_{\rm top}$. While the MLT value stays practically constant, the FST value jumps by ~ 0.1 dex, due to the sharp decrease of $v_{\rm conv}$ at the boundary in the latter case. For this reason, we established to compute $\tau_{\rm glob}$ by taking as top value for the integral $(R_{\rm top} - R)/H_{\rm p}^{\rm top} = 0.1$.

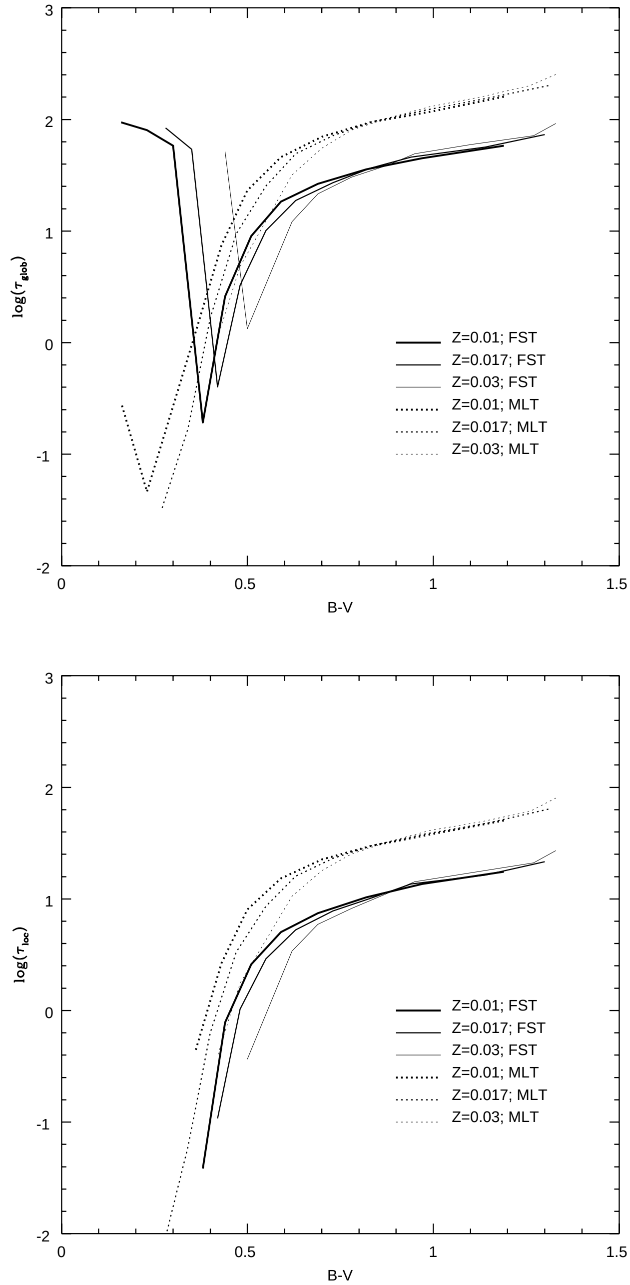

Fig. 2. Global (top) and local (bottom) convective turnover times (in days) vs. the main sequence $B-V$ color, for MLT models (dashed) and FST models (solid lines).

The second possibility, already adopted by Gilman (1980), is to compute a "local" turnover time:

$$\tau_{\rm loc} = \frac{\alpha H_{\rm p}}{2 v_{\rm conv}} \tag{2}$$

where $\alpha = 1.9$ is the mixing-length ratio, and $v_{\rm conv}$ is the convective velocity taken at $\alpha H_{\rm p}/2$ above the convective bottom. Kim & Demarque (1996) have shown that – in MLT models – this quantity has the same dependence on effective temperature and stellar age as $\tau_{\rm glob}$, except for a scaling factor.

Figure 2 shows $\tau_{\rm glob}$ and $\tau_{\rm conv}$ for three different metal abundances, and for the two convective models. The values are plotted as a function of the $B-V$ color, obtained from $T_{\rm eff}$ by using the Castelli et al. (1997) color-$T_{\rm eff}$ conversions. The behavior is substantially similar for the FST and MLT models, although the absolute value of the FST turnover times are smaller than in the MLT case by a factor ~ 2, for the reasons explained above. The novel computations of the convective turnover times for FST models are reported in Table 1, for the three metallicities and the two isochrones here considered.

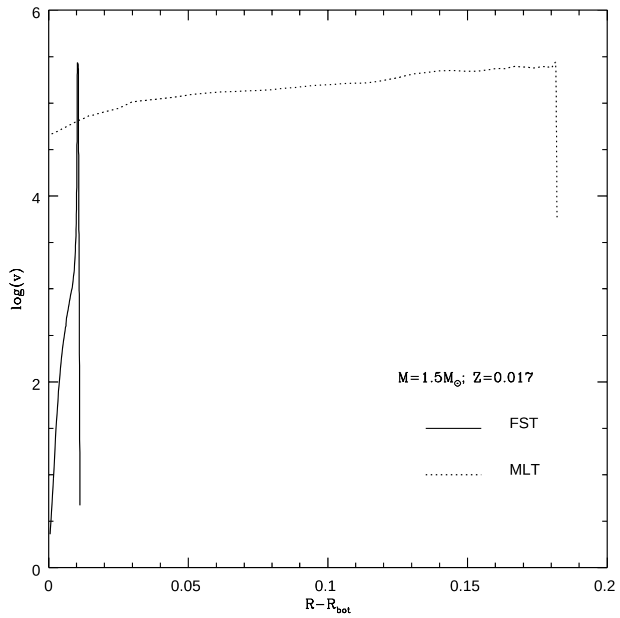

Fig. 3. Logarithm of the convective velocity (in cm s^{-1}), vs. the distance from the base of the convective region, in units of 10^{10} cm, for an MLT and an FST model of $1.5M_\odot$. The FST convective region (solid line) is very small compared to the MLT region (dotted).

Note that $\tau_{\rm glob}$ is characterized by a minimum value at $B-V \approx 0.4$–0.5, corresponding to the locus of the onset of efficient convection, for different values of the stellar metallicity. The sharp rise of $\tau_{\rm glob}$ for $B-V < 0.4$, corresponding to the models for stars of $M = 1.4$ and $1.5M_\odot$, is due to the fact that in these stellar models the convective regions are very shallow, and the convective efficiency is very small (notice that $\tau_{\rm loc}$ cannot be computed for these models because the size of the convection zone is smaller than one pressure scale height). Figure 3 shows in fact the logarithm of the convective velocities inside a $1.5M_\odot$ star, vs. the distance from the base of the convection zone: while the MLT model shows a relatively large region ($\sim 1.8 \cdot 10^9$ cm wide), the FST model has a very narrow convection region ($\sim 10^8$ cm), and very low velocities at both boundaries of the convection zone. This is why $\tau_{\rm glob}$ is very large also for stars with $M \gtrsim 1.3M_\odot$. Of course, when $\tau_{\rm glob}$ is interpreted in terms of the Rossby number, this behavior of the convective turnover time is misleading, because very small convective regions may not be able to trigger an efficient magnetic dynamo and hence to produce an important corona.

We have found that $\tau_{\rm glob}$ and $\tau_{\rm loc}$ differ by a scaling factor also in the FST models, due to the following circumstances: (a) the convective velocity profiles, for stars of different masses, are all quite similar, having a sharp rise just above the base of the convection zone (where $v_{\rm conv}$ is low), followed by a much slower rise in the rest of the convective envelope; (b) the sizes of the convective regions are also similar, being $\approx 3H_{\rm p}$, except for stars with very shallow convection zones; (c) then, $\tau_{\rm glob}$ is dominated by the time needed by a gas blob to crawl the initial few tenths of $H_{\rm p}$ from the convection bottom, and $\tau_{\rm loc}$ is computed in deep layers ($1H_{\rm p}$ above the bottom) where $v_{\rm conv}$ has already reached a relatively large value, effectively comparable to a velocity averaged over the whole convective region. The above qualitative explanation holds also in the MLT case, and the final scaling of $\tau_{\rm glob}$ and $\tau_{\rm loc}$ turns out to be weekly dependent on the precise location where the latter is evaluated, provided that such a location is not too close to any of the boundaries of the convection zone. For ease of comparison with previous works, we have then adopted $\tau_{\rm loc}$ as a measure of the characteristic convective turnover time, in the rest of our work.

3. Empirical vs. theoretical turnover times

Before addressing the issue of metallicity and age effects on the properties of the stellar convection zones, and hence on the Rossby number, we have performed a comparison between the model-based turnover times and the empirically-derived ones. The aim of this comparison is to clarify, in a quantitative manner, the possible discrepancies between a purely theoretical quantity describing the convection characteristics, and a color-dependent quantity related to an empirical parameterization of the magnetic dynamo efficiency. The key point is that while the empirical turnover time is the "correct" dynamo-related parameter which improves the rotation-activity relationship, its determination may be affected by observational biases, and its dependence on stellar parameters other than a color index may be difficult to discover. If this parameter has indeed to do with stellar convection, then theoretical stellar models can be of valuable help to investigate on any such dependence.

The widely used semi-empirical formula by Noyes et al. (1984) for the convective turnover time as a function of the $B-V$ color, was derived from a sample of 41 late-type dwarfs covering the $B-V$ range $0.4 - 1.4$, but with only 5 stars having $B-V > 1$. Instead, Stepien (1994) obtained discrete values of his empirical turnover times in 7 bins covering the range $0.5 < B-V < 1.6$, but again with very few stars redder than $B-V = 1.2$; using both chromospheric and coronal data, he obtained a curve flatter than the Noyes' one for $B-V > 0.8$. In Fig. 4 we show these two $\log \tau_{\rm conv}$ vs. $B-V$ color curves, and

Table 1. Convective Turnover Times in FST convection models

Mass	Z=0.01			Z=0.017			Z=0.03		
$M/M_\odot$	B-V	$\tau_{\rm loc}$ (days)	$\tau_{\rm glob}$ (days)	B-V	$\tau_{\rm loc}$ (days)	$\tau_{\rm glob}$ (days)	B-V	$\tau_{\rm loc}$ (days)	$\tau_{\rm glob}$ (days)
				$\log t = 7.3\,{\rm yr}$					
0.6	1.30	42.02	145.71	1.35	52.90	179.26	1.34	65.09	215.52
0.7	1.22	31.15	108.02	1.31	41.07	139.15	1.33	51.70	175.18
0.8	1.00	22.57	80.07	1.21	31.15	121.20	1.29	40.13	139.15
0.9	0.82	14.57	52.90	0.99	22.05	80.07	1.17	30.44	108.02
1.0	0.64	7.30	27.76	0.85	15.26	55.40	1.01	23.63	83.85
1.1	0.49	2.21	7.47	0.70	8.58	32.62	0.92	16.73	91.94
1.2	0.45	1.18	4.20	0.56	2.65	9.63	0.80	10.32	40.13
1.3	0.39	0.09	0.62	0.52	1.92	6.07	0.65	4.11	15.61
1.4	0.30	—	37.45	0.45	0.29	0.25	0.60	2.91	19.21
1.5	0.23	—	98.51	0.35	—	48.25	0.57	1.79	6.22
1.6	0.16	—	87.80	0.28	—	62.16	0.49	0.14	0.92
				$\log t = 8.5\,{\rm yr}$					
0.6	1.19	17.52	58.01	1.30	21.55	73.03	1.33	27.13	91.94
0.7	0.97	13.60	45.03	1.14	16.35	56.69	1.27	21.06	71.37
0.8	0.82	10.32	35.77	0.94	13.60	46.08	1.10	17.12	59.36
0.9	0.69	7.47	26.51	0.83	10.08	36.60	0.95	14.24	49.37
1.0	0.59	5.05	18.34	0.73	7.83	27.13	0.88	11.31	39.22
1.1	0.51	2.59	8.98	0.63	5.29	18.77	0.78	8.19	30.44
1.2	0.44	0.78	2.59	0.55	2.91	10.08	0.69	5.94	21.55
1.3	0.38	0.04	0.19	0.48	1.03	3.26	0.62	3.42	12.12
1.4	0.30	—	58.01	0.42	0.11	0.40	0.55	1.46	4.94
1.5	0.23	—	80.07	0.35	—	54.14	0.50	0.37	1.33
1.6	0.16	—	94.08	0.28	—	83.85	0.44	—	51.70

the related differences in log $R_{\rm O}$. Inspection of this figure reveals that the deviation in $R_{\rm O}$ is at most by a factor ~ 1.4 at $B-V \simeq 1.5$. In practice, the empirical $\tau_{\rm conv}$ is still poorly constrained by the available data for stars redder than $B-V \sim 1.3$.

In Fig. 5 we show the comparison between the empirical turnover times previously described and the model values, computed for main-sequence stars of age $\log t = 8.5\,{\rm yr}$ with solar metallicity. We find that the dependence of the former one on $B-V$ is everywhere less steep than predicted for $\tau_{\rm conv}$ by the models. We stress that, in this comparison, the absolute values are less important than the slopes, because any vertical shift of the curves would affect only the normalization (rather than the functional form) of the activity vs. Rossby number relationship. The largest deviations occur for the bluer stars ($B-V \lesssim 0.5$, corresponding to solar-metallicity stars of $\sim 1.4{\rm M}_\odot$), where very short turnover times are predicted due to the small extention of the convection zone (Sect. 2).

From inspection of Fig. 5 we conclude that, except for the stars blueward of $B-V \sim 0.5$, the relative deviation of the theoretical turnover times from the empirical ones is contained within a factor ~ 2.5, for both the MLT and the FST model. Major differences exist between the two models regarding the location of the onset of an efficient convection (Canuto & Mazzitelli 1991, 1992), so that we defer to a future work a detailed comparison between model predictions and observational evidences of magnetic activity in stars with very shallow convection zones.

As a final note, we stress that empirical turnover times are not necessarily better than the model-predicted ones in the task of providing a tight activity vs. Rossby number relationship, for stars in a wide range of spectral types. We have recently checked this behavior for a sample of 37 Hyades cluster members, with $0.61 \le B-V \le 1.63$, having known rotational periods or surface velocities (Pizzolato et al. 2000): by performing power-law correlations between their X-ray to bolometric flux ratio and the Rossby number, the latter evaluated using either theoretical or empirical turnover times, we have found that the theoretical convective times yield a better correlation (i.e. smaller residuals) than the Noyes et al. (1984) $\tau_{\rm conv}$ values; on the other hand, similar correlation results have been obtained by adopting either the MLT or the FST model values.

4. Metallicity effects on $\tau_{\rm conv}$

The metal abundance determines the gas opacity in stellar interiors and hence the conditions for convective instability. For a *star of given mass and age*, higher metallicities are expected to produce deeper convection zones,

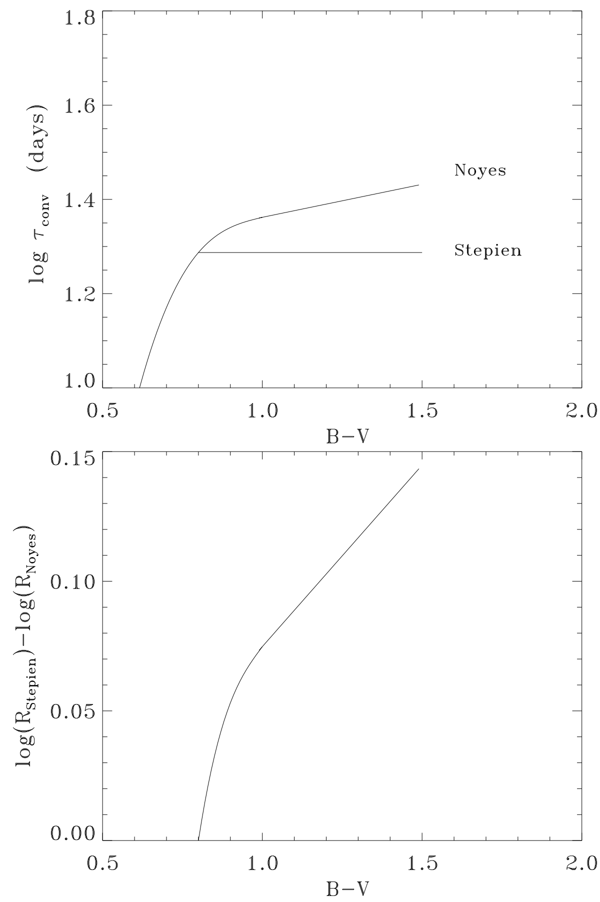

Fig. 4. Deviations in the log Rossby number (bottom) due to the Noyes et al. and Stepien empirical way to evaluate the convective turnover time (top).

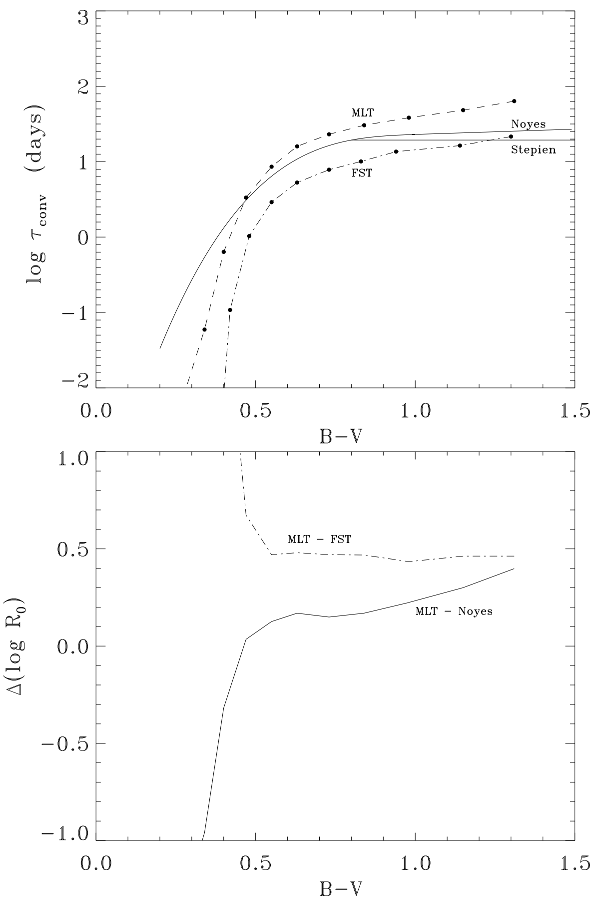

Fig. 5. Comparison between empirical turnover times and model-determined $\tau_{\rm conv}$ (top). Differences in the logarithm of the Rossby number are also shown (bottom) between the MLT and FST convection models (at $Z = Z_\odot$ and log(age)=8.5), and between the MLT model and the Noyes et al. (1984) semi-empirical values.

but the behavior of the convective velocities and hence of the turnover times is less obvious. A crucial effect is also played by the dependence of the $B - V$ colors on the metallicity: in fact, stars are observationally selected and compared in limited color ranges, rather than in mass ranges.

In Fig. 6 we show $\tau_{\rm conv}$ curves vs. mass, computed with either the MLT or the FST model, for stars with three different metallicities, $Z = 0.01, Z = 0.017 = Z_\odot$, and $Z = 0.03$. The effect of the metallicity on the Rossby number, assuming the same rotation period in all cases, is shown in the lower panel of the same figure. For stars of $0.6{\rm M}_\odot$, the convective time scales increase by a factor ~ 2 when Z increases from 0.01 to 0.03, for both convection models, while for stars of $1.3{\rm M}_\odot$ a factor ~ 25 is found with the MLT treatment and a factor ~ 100 in the FST case. In other words, the metallicity effect is in line with the expectation of a more developed convection zone for stars with higher metallicities, and this effect is more pronounced for the more massive stars or when the FST convection theory is preferred.

Figure 7 shows instead the case in which $\tau_{\rm conv}$ is plotted vs. $B - V$ color. Note *the reversed behavior* for the late-F and G stars ($0.5 < B - V \lesssim 0.8$): if the color is kept fixed, the stars with higher metallicities are predicted to have shorter turnover times (and hence, higher Rossby numbers). On the other hand, the effect is almost negligible for the dK stars ($0.8 \leq B - V < 1.3$). This counterintuitive result is explained by noting that stars tend to become redder when the surface gas opacity is increased, and this color shift implies that F and G stars with higher metallicities are in fact more massive (and hence with less deep convection zones) than stars with lower metallicities but the same color.

The expected metal-induced variation of the Rossby number is up to a factor 2.5 (comparying the $Z = 0.01$ and $Z = 0.03$ cases) for late F-type stars, according to the MLT model predictions, and a factor ~ 4 following the FST approach. A change by less than a factor 1.25 is predicted for stars redder than $B - V \sim 0.7$. Since larger Rossby numbers are observationally associated to less active stars, the above results suggest lower coronal emission levels for solar-type stars with higher metallicities.

However, there is a second metallicity-dependent effect to take into account, beside the one on the characteristics of the convetive envelope, when trying to predict the

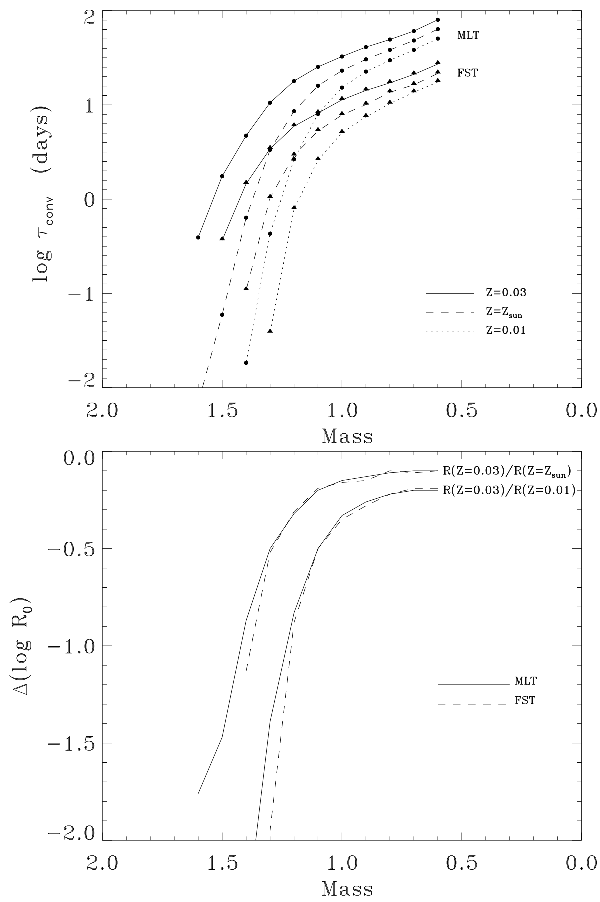

Fig. 6. Top: Convective turnover time vs. stellar mass (in solar units) predicted by MLT and FST models, at log(age)=8.5, for three values of the metallicity. Bottom: Variations in the logarithm of the Rossby number due to the different metal contents.

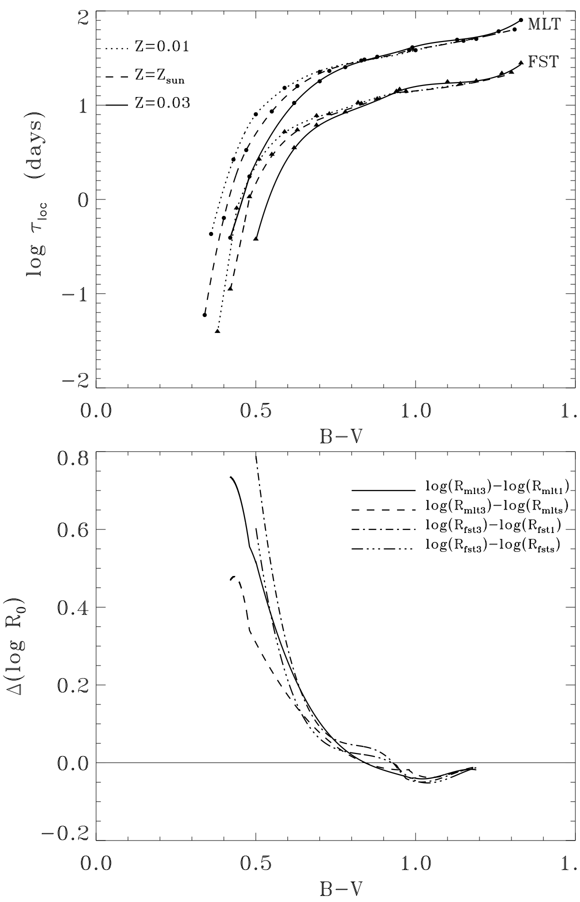

Fig. 7. Top: Convective turnover time vs. $B-V$ color index, predicted by our stellar models, at log(age)=8.5, for each assumed metallicity. Bottom: variations of the Rossby number due to the different metal abundances. The labels are MLT and FST for the two convective theories, the number 1 indicates the Z=0.01 case, the number 3 stands for Z=0.03, and the label "*s*" is for the solar metallicity models.

overall influence of metal abundances on the observed X-ray emission level, namely a possible change of the plasma emissivity in corona. This effect will be taken into account in the test presented in Sect. 6.

5. Age influence on $\tau_{\rm conv}$

Stellar age is another, obvious parameter which the characteristics of subphotospheric convection depend upon, and hence potentially relevant when $\tau_{\rm conv}$ need to be evaluated. In fact, all studies of the magnetic activity vs. Rossby number relationship for main-sequence stars, including those for stars in young stellar clusters (e.g. Randich 1999), have simply ignored any age-dependence of the convective turnover time. In this section we show how much spurious spread could be introduced in the above relationship if the dependence of $\tau_{\rm conv}$ on age is neglected; to this aim, we have considered the predictions of stellar models at two different evolutionary times, $\log t = 7.3$ and $8.5\,$yr, selected as anticipated in Sect. 2.

In Fig. 8 we compare the model predictions for stars of solar metallicity. The younger stars redder than $B-V \approx 0.7$ tend to have longer convective times than the older stars of the same color, due to deeper convection zones in the evolutionary stages preceeding the arrival on the ZAMS. The implied color-dependent variation of the Rossby number, due to the stellar ageing, increases from zero at $B-V \sim 0.5$ to a factor ~ 2 for the reddest stars considered ($B-V \sim 1.3$). This prediction is independent from the convection theory assumed, and we have also verified that the behavior does not change appreciably if metallicities other than the solar one are considered.

We conclude that the present age effect may be important when comparying dK and dM stars of different ages. However, the influence on the expected coronal emission level cannot be predicted easily, because a major age effect to be taken into account is the one on stellar rotation (i.e. on the numerator of $R_{\rm O}$), which - for main-sequence stars - is mainly due to a wind-driven magnetic bracking which in turn depends on the level of surface magnetic activity.

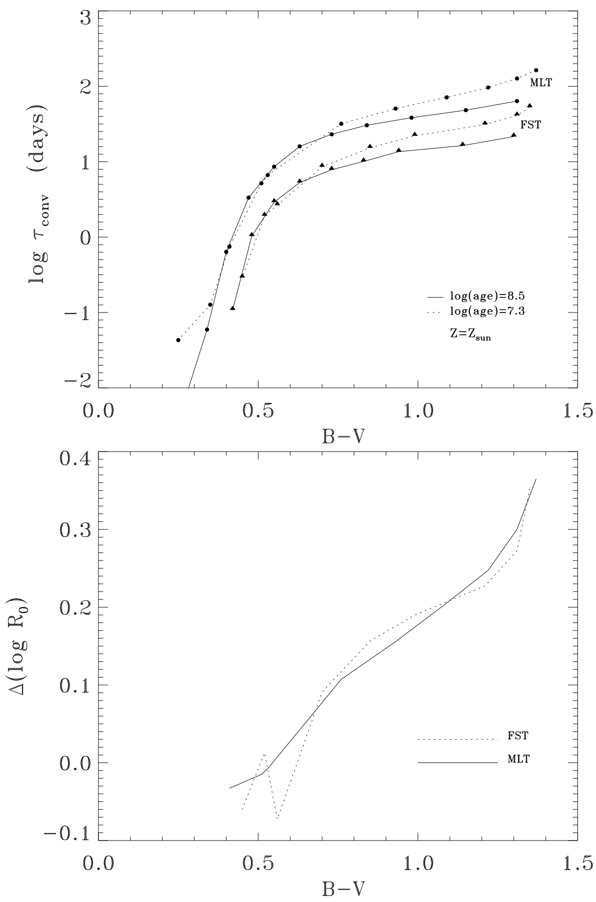

Fig. 8. Convective turnover time vs. $B-V$ color (upper panel), as predicted by the MLT and FST stellar models, for the two evolutionary ages indicated, and the corresponding variation in $\log R_{\rm o}$ (bottom panel).

6. Testing metallicity and age effects on the X-ray emission level

In the previous sections we have shown that, in a main sequence low-mass star, the convective turnover time depends, not only on stellar mass, but may depend also on metallicity and age. Both effects are largely independent from the specific convection model assumed and, while abundance effects are relevant for stars in the $0.5 \leq B-V \leq 0.7$ range, the age influence is likely important only for stars with $B-V > 0.7$.

To verify that these effects produce an observable influence on stellar dynamo, we propose two tests that make use of X-ray observations of selected open clusters, and in particular of the median values of their maximum likelihood X-ray luminosity functions[1], for stars of a given spectral type. The median values of homogeneous samples, like open clusters, allow us to reduce the uncertainties related to individual stellar properties and to coronal variability.

[1] The maximun likelihood luminosity function takes into account both of detections and upper limits measured on the star positions.

To test the effect of the metal abundance we have compared the X-ray luminosity level of dG and dK stars in three open clusters with similar age, but different metallicity. To test the age effect, we plan to compare clusters with similar metal abundance but different age, in a future work.

In order to test our expectation of a metal abundance effect on X-ray luminosity, we compare the X-ray luminosity levels of three clusters: the Pleiades, Blanco 1, and NGC 2516. These three clusters have age $\sim 10^8$ years but different metallicity: The Pleiades are the best studied cluster, having well known membership and solar abundance (Boesgaard & Friel 1990). We use the ROSAT determination of the maximum likelihood X-ray luminosity function (Stauffer et al. 1994; Micela et al. 1996). Blanco 1 has metal abundance twice the solar one (Edvardsson et al. 1995), its membership has been recently redetermined using GSC-II data (Hawkins et al. in preparation), and new X-ray luminosity functions are now available (Pillitteri et al. in preparation), based on the astrometric membership and ROSAT HRI data (Micela et al. 1999). NGC 2516 has a metal abundance about half the solar one (Cameron 1985; Jeffries et al. 1997, 1998). For this cluster only a photometric membership is available (Jeffries et al. in preparation) while Chandra and XMM-Newton X-ray luminosity functions have been already published (Harnden et al. 2001; Sciortino et al. 2001).

We assume that the rotation distributions of the three clusters are the same. In first approximation this is a reasonable hypothesis since the clusters have same age, but in practice this assumption has not been observationally tested because extensive rotational surveys of NGC 2516 and Blanco 1 have not been carried out yet.

To the aim of our test, we have explicitly taken into account also the influence of the metallicity on the radiative losses of the coronal plasma. In fact, at typical coronal temperatures ($T_{\rm c} \approx 10^6$–10^7 K), X-ray radiation is dominated by emission lines from highly-ionized atomic species, and hence it depends on the abundances of heavy elements in corona.

In Fig. 9 we show the plasma emissivity, $P(T_{\rm c})$, vs. coronal temperature for an optically-thin plasma as predicted by the Raymond & Smith (1977) code, for different metal abundances (the abundance ratios are kept fixed to the solar values). In the same figure, we also show the variation of the emissivity for metallicity changes by a factor 2 or 4: if we assume the same plasma emission measure in stars with different metallicities, the coronal emission is expected to depend almost linearly on Z, for a fixed coronal temperature in the range 10^6–10^7 K. More precisely, for a coronal temperature of 10^6 K, the emissivity increases by a factor ~ 1.9 when Z goes from half-solar to the solar value, and by a factor ~ 3.6 in going from $Z_\odot/2$ to $2Z_\odot$; at $T_{\rm c} = 10^7$ K, the change is by factors 1.7 and 2.7, respectively. For hotter plasmas, the increase in emissivity is smaller, because of the reduced contribution of line emission to the radiative loss function.

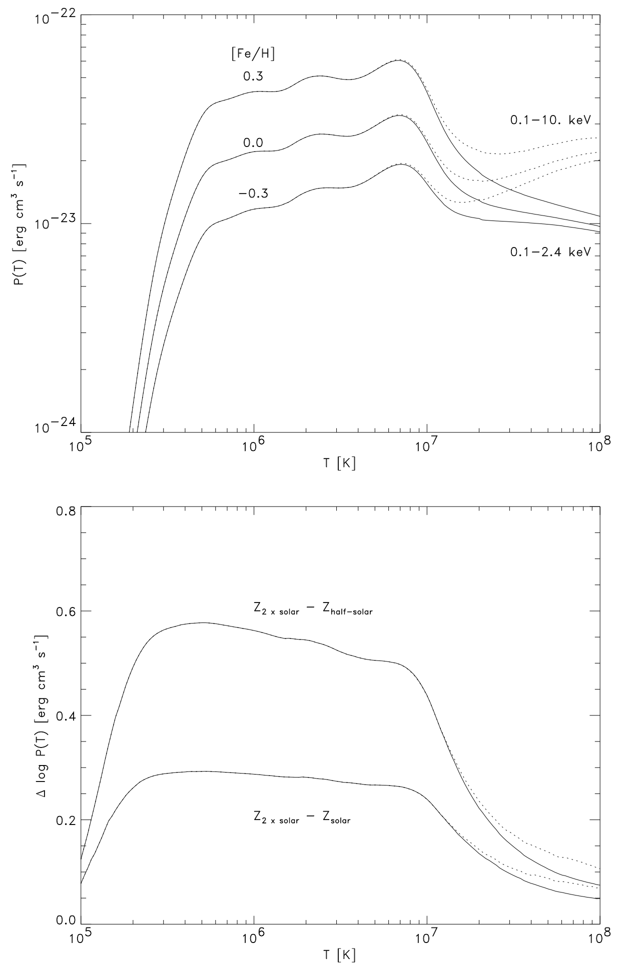

Fig. 9. Effect of the metallicity difference on the emissivity of the coronal plasma: the upper panel shows the plasma emissivity vs. temperature assuming three values of metallicity, parametrized by [Fe/H]; the lower panel shows the logarithm of the emissivity ratio in going from lower to higher metallicities. Solid and dotted lines are used for the two indicated energy bands, in which radiative losses have been computed.

Note that this effect on the coronal emission is in the reverse sense with respect to the one, in the 0.5–0.8 B-V range, based on the model-predicted dependence of $\tau_{\rm conv}$ on Z and on the empirical relationship between coronal activity and Rossby number. In other words, late-F and G stars with the same color but higher Z are expected to have a less efficient convection (shorter turnover times) and hence a less efficient magnetic dynamo and lower coronal emission, but - on the other hand - their coronal plasma is expected to radiate more efficiently, for the same emission measure. Which one of the two effects dominates depends critically on the stellar $B-V$ color (because of the steep dependence of $\tau_{\rm conv}$ on $B-V$), and less importantly also on the coronal temperature and on the emission vs. Rossby number dependence.

Instead, for the dK stars ($B-V > 0.8$), the metallicity effect on the plasma emissivity appears to be always the dominant one, because $\tau_{\rm conv}$ depends very little on Z. We note in passing that our assumption of equal metal abundances in corona and in the stellar interior may not be valid in general: indications exist that coronal abundances in very active stars are lower than the photospheric ones. Hovever, given the lack of a clear picture of this phenomenon, we consider our assumption still reasonable.

In order to carry on our test, we have then assumed that the distribution of rotation periods is the same for the stars in the three clusters we are considering, and that the functional dependence of the X-ray emission level on the Rossby number can be described by following functional form:

$$L_{\rm x} \propto R_{\rm O}^{-a} \propto \tau_{\rm conv}^{a} \tag{3}$$

where the exponent a has been determined in previous studies to be in the range 1–2 (Micela et al. 1984; Maggio et al. 1987; Dobson & Radick 1989; Hempelmann et al. 1995).

We assume a coronal temperature T=10^6-10^7 K, independently of metallicity, and consider separately dG (defined as stars in the range $0.5 \leq B-V \leq 0.8$), and dK stars ($0.8 \leq B-V \leq 1.4$). In the case of dG stars the metallicity effects on $\tau_{\rm conv}$ and on radiative losses tend to compensate, while in the case of dK stars only the effect on the radiative losses is relevant, hence the overall dependence of $L_{\rm x}$ on metallicity will be steeper for dK stars than for dG stars.

We have evaluated the expected variations for $Z = 0.001$, $Z = Z_\odot = 0.0017$, and $Z = 0.003$, corresponding to the metallicity of the three clusters. The results, together with the median values of the X-ray luminosity functions and associated statistical uncertainties[2] are shown in Fig. 10 and 11. All the values in the plot are normalized to the Pleiades value, and we estimate the relative variations of the low and high metallicity clusters with respect to the Pleiades.

In the case of the dK stars (Fig. 10) the only important effect is that due to the radiative losses; the plot shows the expected values, corresponding to the different metallicities, compared with the medians of the $L_{\rm x}$ distributions of our clusters. The dashed area in the figure shows the uncertainty due to the range of possible values of coronal temperature ($10^6 \leq T \leq 10^7$ K). We note that, while the expectations corresponding to NGC 2516 and the Pleiades, well predict the observed difference between the two clusters, the high metallicity prediction (corresponding to the Blanco 1 point), predicts a value significantly higher than the observed one.

The case of the dG stars is more complicated because both $\tau_{\rm conv}$ and the the radiative losses are affected by the metal abundance. Fig. 11 shows our predictions with the uncertainty due to our choice of each relevant parameter.

[2] The formal statistical uncertainties at the 68% confidence level on the median values have been evaluated with a bootstrap method (Schmitt 1985). Note that the uncertainty for the G stars in Blanco 1 is relatively large because the X-ray luminosity function barely reaches the median value.

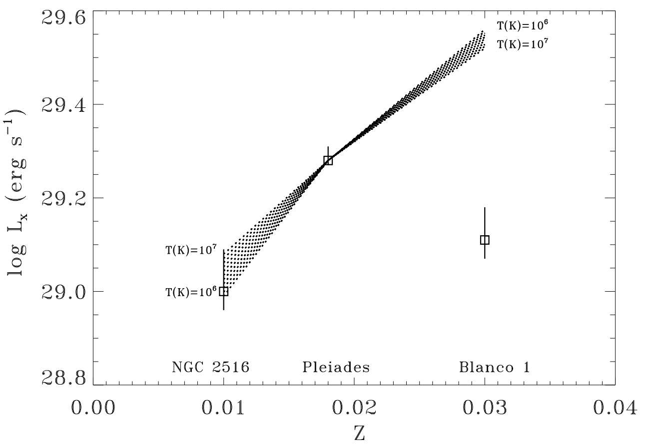

Fig. 10. Comparison between the median X-ray luminosity of dK-type stars (squares) for the stellar clusters NGC 2516 (Harnden et al. 2001), Pleiades (Micela et al. 1996) and Blanco 1 (Pillitteri et al., in preparation). The error bars indicate the formal statistical uncertainties on the median values. The dashed lines indicate the expected trends due to the effect of different metal abundance on radiative losses, assuming coronal temperature spanning the $10^6 - 10^7$ K range.

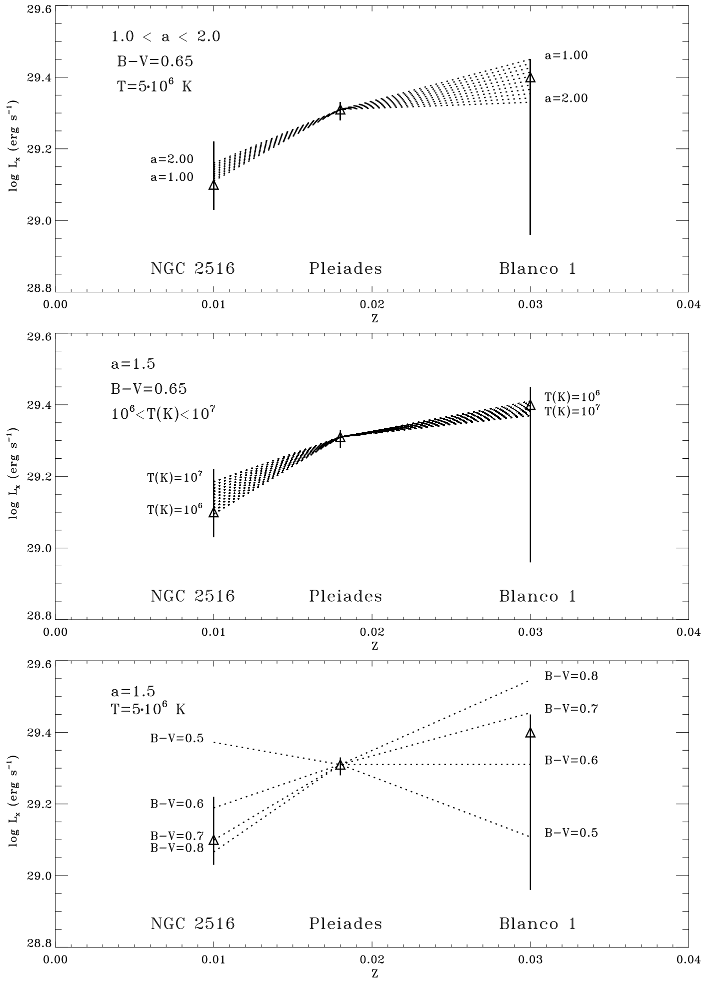

Fig. 11. Comparison between the median X-ray luminosity and related uncertainties (see text) of dG-type stars (triangles) for the stellar clusters NGC 2516 (Harnden et al. 2001), Pleiades (Micela et al. 1996) and Blanco 1 (Pillitteri et al., in preparation). The dashed area indicates the expected trends due to the total effect of metal abundance on radiative losses and on convective turnover time. The upper panel shows the area allowed by the uncertainty in the index of the power-law in Eq. 3. The central panel shows the area allowed by differences in coronal temperature, and the lower panel indicates the spread due to the $B-V$ color range of the sample stars.

In particular the upper panel shows, for stars with $B-V=0.65$, the uncertainty due to the choice of a in eq. 3, assuming values in the 1-2 range. The largest sensitivity to the value of a is obtained in comparing solar and high-metallicity stars, where $\tau_{\rm conv}$ is particularly dependent on abundance, introducing a difference between the predicted $L_{\rm x}$ ranging from 0.05 dex ($a=2$) to 0.15 dex ($a=1$). The observed difference between $0.5Z_\odot$ and $Z_\odot$ is less sensitive to a, and it goes from -0.15 to -0.20 dex.

The central panel shows the error due to uncertainty in coronal temperature, analogously to the case of dK stars, while the lower panel shows the spread due to the $B-V$ color range. In this last panel, it is interesting to note that in going from stars with $B-V=0.5$ to stars with $B-V=0.8$ the trend of our predictions versus metallicity changes the sign of the slope. This is due to the rapid dependence of $\tau_{\rm conv}$ on metallicity in the above color range (see Fig. 7). In particular, for stars at $B-V=0.5$, the (negative) effect in high metallicity stars on $\tau_{\rm conv}$ is much larger than the (positive) effect on radiative losses, producing a net decrease of the predicted $L_{\rm x}$ for increasing metallicity; at $B-V=0.6$, the two effects compensate more or less exactly, while for later spectral types, the effect of radiative losses dominates.

All the uncertainties and spread described above are consistent with the data of the dG stars in the three clusters, while for the dK stars the agreement between predictions and observations is restricted only to the low-metallicity and solar-abundance stars. The dK stars at high metallicity, namely those of the Blanco 1 cluster, appear too underluminous in X-rays with respect to our expectations. The phenomenon known as "saturation" is not relevant for this case, because – for dK stars – it implies a threshold X-ray luminosity of $\approx 10^{30}\,{\rm erg\,s^{-1}}$, well above the median value found for Blanco 1 members. Other possible effects we can invoke are heavy contamination of the sample due to field stars and/or a rotational distribution very different from that of the Pleiades (with very few fast rotators present in Blanco 1). The latter possibility appears to us the most plausible at the moment. In this respect, we note that the rotational evolution of a star could depend on metallicity as a second-order effect, because different levels of magnetic activity could imply also a different efficiency of the magnetic braking. New rotational data are needed to confirm our hypothesis that the distributions of the rotational velocities are indeed similar for the three clusters.

7. Summary and conclusions

In this paper we have critically reviewed the interpretation of the Rossby number, used to understand the stellar activity properties. In particular, we have compared empirically defined convective turnover times (Noyes et al. 1984; Stepien 1994), usually adopted to estimate the Rossby number, with model-predicted turnover times, computed assuming either the classical MLT treatment of convection or the more advanced FST approach (Ventura et al. 1998).

We have verified that turnover times predicted by the two models differ in normalization, but not in shape, except near the masses where the onset of efficient subphotospheric convection is expected. Furthermore, we have checked that turnover times evaluated locally (at one pressure scale height above the bottom of the convection zone) scales with the global ones (integrating over the whole depth of the convection zone), both for the MLT models and for the FST models; this finding makes the use of the local turnover times quite robust with respect to the assumption of the (uncertain) location where dynamo action takes place in the stellar interior, and demonstrates that local and global turnover tims are equivalent in studying the relationship between magnetic activity level and Rossby number, irrespective of the adopted convection theory. In present paper, we have then used the local turnover times, $\tau_{\rm conv}$.

We have also verified that the differences between empirical and model-predicted turnover times are relatively small with respect to the variations introduced in $\tau_{\rm conv}$ by changes in stellar properties, such as metallicity or age. Hence, we are confident that we can use stellar models to explore the dependence of the Rossby number on these stellar properties.

The sensitivity of $\tau_{\rm conv}$ and of the color-mass relationship to metallicity makes critical the knowledge of this latter quantity in attempting to assess empirically any correlation between activity indicators (e.g., the coronal X-ray luminosity, $L_{\rm x}$) and Rossby numbers, in order to minimize the spread introduced by an incorrect evaluation of $\tau_{\rm conv}$ in different metal abundant stars. This effect could be as large as a factor three in dG disk population stars.

The maximum sensitivity of the $\tau_{\rm conv}$ to variations of metal abundance is obtained at dG stars, that could be a good diagnostics to explore this effect, while the maximum sensitivity to age effects is confined to dK stars.

In order to explore the metallicy influence on $\tau_{\rm conv}$, we have compared the median $L_{\rm x}$ observed in dG and dK stars of three open clusters with similar age and different metallicity: NGC 2516, the Pleiades, Blanco 1. In this comparison we have taken into account also the effects of metallicity on coronal radiative losses. Our expectations for dG stars are fully consistent with data, while for dK stars we are able to well predict differences in X-ray emission between the low and the solar metal-abundance clusters, while our expectations overestimates the $L_{\rm x}$ value of the high metallicity cluster Blanco 1.

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