Statistical Properties of Wavelet Transforms Applied to X-Ray Source Detection

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ABSTRACT We have developed a method for detection of sources in X-ray images, based on wavelet transforms (WT). After having computed the WT of an image, for various values of the wavelet scale parameter, candidate sources are selected as local maxima in the wavelet-transformed image. The reliable discrimination between true sources and random background fluctuations requires a detailed knowledge of the distribution of WT values arising only from background noise. The shape of this distribution may be very different from a gaussian, especially in the limit of few counts per image resolution element, which is the case in most X-ray images.
By means of both analytical means and numerical simulations, we have therefore studied the WT probability distribution for a wide range of background density values. This enables us to derive thresholds for source detection in the WT images, for a range of confidence levels. These detection thresholds are now being used in our detection algorithm with good results.

1 Description of the Method

With the advent of new X-ray detectors having higher spatial resolution, and therefore less background photons per resolution element, the case of sparse-photon X-ray images is becoming ever more common. Under such circumstances, it may be possible to detect sources showing just a few imaged photons, provided that the analysis method is powerful enough. To this aim, we have recently developed a source detection method, based on wavelet transforms (WT), for images produced by photon-counting detectors, that has been tuned accurately in order to fully exploit its capabilities.

Our algorithm selects sources as spatial maxima in the WT of the analyzed image (at various spatial scales a) rather than in the image itself. The statistical distribution of photons in a pixel of the original image (expected to be locally a Poissonian) is thus converted into another distribution $P(w)$ of values w in the wavelet-transformed image. Therefore, in order to discriminate efficiently WT maxima due to real sources from those arising from background fluctuations we must know in detail the distribution $P(w)$ of WT values due only to background fluctuations.

2 The Distribution $P(w)$

Adopting as a generating wavelet the so-called 'mexican hat' function, best suited for our problem, we find that the distribution $P(w)$ has a complicated shape, that depends on the background intensity. More specifically, it can be seen that the shape of $P(w)$ depends only on the product $q = b \times a^2$, namely the number of background photons per squared wavelet scale, so we denote this distribution as $P_q(w)$. As the number q becomes lower, the image photons are more sparse, their statistics is more markedly Poissonian, and the WT distribution is increasingly different from a gaussian (we call this the 'discrete photon limit'). Instead, as q rises (say $q \geq 300$) the distribution $P_q(w)$ approaches slowly a gaussian, as it can be shown analytically, since the distribution of photons in the analyzed image now approaches a gaussian (the 'continuous limit'). Such an analytic description has not however been found for the discrete photon limit.

Since current X-ray detectors have many thousands independent resolution elements, source detection involves a comparably large number of 'trials', whichever method one is using. Therefore, in order to keep the number of expected spurious detections reasonably low (1-2 per field), the appropriate detection threshold must be derived from the extreme positive tail of the distribution $P_q(w)$, that must be therefore known in detail. Since in the discrete photon limit we have no analytic expression for $P_q(w)$, we have computed it numerically, through large sets of Montecarlo simulations (10^7 realizations for each background value), needed to describe accurately even the tail of $P_q(w)$. As a result, we have derived detection thresholds (in the WT space) for a large range of background values, and for any desired confidence level (namely, the probability that a given detection is due to a background fluctuation). Eventually, since our algorithm involves a cross-identification between sources detected at various scales a, we have applied the entire algorithm to simulated background images, to derive the overall number of spurious detections corresponding to a chosen confidence level.

3 Conclusions

Currently, our algorithm works successfully on ROSAT PSPC and HRI images. Especially in the center of ROSAT HRI images, where spatial resolution is highest, the number of background photons per resolution element (of size given by the PSF FWHM) is very low, and the shape of the WT at small scales a is most different from a gaussian. The present study, moreover, will be even more relevant for the analysis of X-ray images with even higher resolution, (e.g. using the AXAF ACIS and HRC), where we expect to detect reliably sources with only a handful of photons. For more details, we refer the reader to Damiani et al. (1996; recently submitted to ApJ).