CLUSTER MEMBERSHIP DETERMINATIONS FROM PROPER MOTION SURVEYS

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ABSTRACT. The advent of large telescopes such as Keck has made possible high resolution, detailed spectroscopic studies of faint, low mass stars in clusters. This new capability has emphasized the need for means to identify faint cluster members. Although the identification of the more luminous stars of a cluster is straightforward, the identification of the low-luminosity members in an unbiased way is difficult, and usually makes use of the fact that cluster members share nearly a common space motion. The most efficient means of finding low-luminosity members is the measurement of proper motions. Various techniques have been used to assign proper motion cluster membership probabilities to stars in the region of a cluster. These techniques are reviewed, as well as the reliability and appropriateness of the methods. Recent surveys and future prospects are discussed. A compilation of recent determinations of cluster membership studies is given.

1. Introduction

The determination of proper motions of stars in the vicinity of star clusters has a long history, beginning with Pritchard's (1884) study of the Pleiades. Vasilevskis (1962) reviewed the work to 1962 and van Leeuwen (1984) through 1983. There are several reasons for determining proper motions of stars in the regions of clusters. Foremost is the determination of cluster membership. Determination of membership is necessary to investigate the spatial distribution of cluster stars, study the morphology of the color-magnitude diagram, investigate the luminosity and mass function, and, increasingly more important, provide observing lists of faint cluster members for spectroscopic observation on large telescopes. In addition to membership studies, proper motions of high precision can be used to determine the internal motions of cluster stars, and proper motions in a well defined reference frame can be used to determine the space motion of the cluster as a whole. In this review, we are interested primarily in the use of proper motions to determine cluster membership.

We recognize open clusters because the areal density of bright cluster stars near the cluster center is much higher than that of field stars. However, for faint stars the areal density of field stars exceeds that of cluster members, sometimes by two or three orders of magnitude. This is demonstrated in Fig. 1 which presents separately the distribution of bright and faint stars in the vicinity of the cluster M25. The cluster is clearly evident in the distribution of bright stars, but not noticeable in the distribution of faint stars.

The velocity dispersion of stars in an open cluster is generally quite small, more than an order of magnitude less than the field star velocity dispersion. For practical

Fig. 1. The distribution of stars brighter than V=11 (left) and fainter than V=12 (right) in the vicinity of M25.

purposes, the cluster members have the same space motion, and, for clusters of small angular extent, the same proper motion. Measurement of the proper motion of stars in the vicinity of a cluster allows one to pick out stars sharing the same motion, and are thus likely cluster members. Procedures for assigning probabilities of membership based on proper motion are described below.

Their are two advantages for using proper motions to determine cluster membership. One, the method is basically unbiased. Two, it is very efficient. Providing an early source of positions is available, one can observe all stars simultaneously (either photographically or with some other imaging device) and one can go very faint with a relatively small telescope. The major disadvantage is the necessity of a source of first epoch positions. For many cluster, such data does not exist, or does not exist for faint stars.

2. Proper Motion Determinations

It is not our purpose here to go into the details of proper motion determination, but only to sketch the important features. Nearly all proper motion determinations of stars in the vicinity of clusters have made use of photographic plates (but this is likely to change). In its simplest form, we have measured (x, y) positions of stars at two epochs. The difference in positions between the same star at the two epochs is the result of the stars proper motion, the error of measurement, and effects due to the telescope(s), the atmosphere, and the measurement process. In general, the difference between the x measures (for example) of a given star will be:

$$\mathrm{x}_2 - \mathrm{x}_1 = \mu_{\mathrm{x}} \Delta \mathrm{t} + \mathrm{f}(\mathrm{x}, \mathrm{y}, \mathrm{m}, \mathrm{c}; \mathrm{a}_{\mathrm{i}}) + \epsilon$$

where μ_x is the proper motion, Δt the epoch difference, and f a function of the measured positions, magnitudes (m), and possibly colors (c), and ϵ the measurement error. We note that in the simplest case,

$$x_2 - x_1 = \mu_x \Delta t + a_1 x_1 + a_2 y_1 + a_3 + \epsilon$$

which accounts for scale changes (a_1), rotation (a_2), and zero point differences (a_3) between the two sets of measures.

Given the measured positions, magnitudes, and colors, the problem is to find the parameters a_i that describe transformation between the measures at the two epochs. Once found, the proper can be computed from

$$\mu_x = \frac{(x_2 - x_1) - f(x, y, m, c; a_i)}{\Delta t}$$

with error $\epsilon/\Delta t$ We note that the problem as formulated is degenerate since, for example, an additive constant to all the proper motions would also be a solution. Some assumption about the proper motions is required. Most often in cluster work, relative proper motions are determined, in a reference frame in which one assumes that the mean proper motion is zero, and there is no overall expansion, contraction, or rotation. One can assume this for all stars, or more commonly, for cluster stars only. In practice, the parameters are most often determined using least squares. Least squares is adequate if cluster stars are used for a reference frame and the errors are normally distributed. The use of cluster members as a reference frame in a small field also greatly simplifies the reduction process, and more easily allows the elimination of systematic effects due to magnitude and optical distortion.

In practice, the modeling is usually much more complicated than indicated above. Moreover, one usually has measures from several plates and more than two epochs. Methods for determining the proper motions under these conditions are discussed by many investigators, including Francic (1989), Jones and Walker (1988), and Hambly *et al.* (1995).

The error of a proper motion depends on several factors, including the epoch difference, the number of plates measured, the scale of the telescope, and the accuracy of the measurement. In particular, the focal length of the telescope, which determines the scale, is important. Vasilevskis (1962) suggested a dependence of precision on focal length and epoch difference of $(\Delta t f^{(1/2)})^{-1}$. However, it appears that for photographic work, the precision varies more nearly as f^{-1}. The precision needed for membership determination depends upon the cluster proper motion and the relative number of cluster and field stars. For nearby clusters (Hyades, Pleiades, Praesepe, α Per), the main emphasis has been on a search for faint members near the brown dwarf limit. Because these clusters are nearby and have a large angular size, one needs to cover a large area, and this has necessitated the use of telescopes of short focal length, which means relatively large errors. This has been compensated for by the relatively large cluster motion. There is also the added complication of geometric effects with clusters of large areal extent, i.e., common space motion does not imply common proper motion. More distant clusters require higher precision because their total proper motion is small. This requires the use of telescopes of longer focal lengths and large epoch differences. For more distant clusters, the emphasis has also been on a search for faint members, but not as faint as for the nearby clusters.

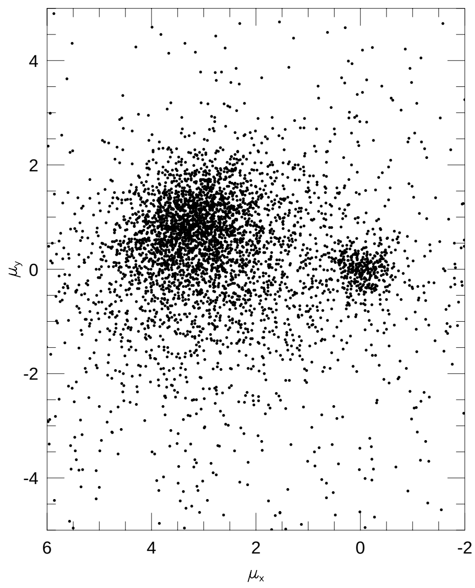

Fig. 2. The proper motion vector point diagram for Praesepe. Units of proper motion are arcseconds per century

3. Proper Motion Membership Probabilities

The concept of proper motion membership probabilities was introduced in a landmark paper by Vasilevskis *et al.* (1958) who proposed looking at the distribution of stars in a proper motion vector point diagram (PMVPD, a plot of μ_x *vs.* μ_y). Fig. 2 show a PMVPD for stars in the region of Praesepe from the data of Jones and Stauffer (1991). In this study, Praesepe stars were used as a reference frame, so that the concentration of stars near (0,0) are Praesepe stars, while the wider concentration near (3,1) are field stars. Praesepe has a fairly large proper motion, so there is a clean separation between the cluster and field.

Suppose one has measured proper motions (μ_x, μ_y) of stars in the region of a cluster. Vasilevskis *et al.* proposed finding separately the distribution of field stars, $\Phi_f(\mu_x, \mu_y)$, and cluster stars, $\Phi_c(\mu_x, \mu_y)$, in a PMVPD. The probability $\mathrm{P}(\mu_\mathrm{x}, \mu_\mathrm{y})$ that a star with proper motion (μ_x, μ_y) is a member is then:

$$P(\mu_x, \mu_y) = \frac{\Phi_c(\mu_x, \mu_y)}{\Phi_f(\mu_x, \mu_y) + \Phi_c(\mu_x, \mu_y)}$$

Vasilevskis *et al.* used a model assuming an elliptical bivariate normal frequency distribution for the field and cluster, although this is easily relaxed to an circular distribution for the cluster. Under these assumptions

$$\Phi_c(\mu_x, \mu_y) = \frac{N_c}{2\pi\sigma_x\sigma_y}\exp - [\frac{(\mu_x - \mu_{x,c})^2}{2\sigma_x^2} + \frac{(\mu_y - \mu_{y,c})^2}{2\sigma_y^2}]$$

$$\Phi_f(\mu_x, \mu_y) = \frac{N_f}{2\pi\Sigma_x\Sigma_y}\exp - [\frac{(\mu_x - \mu_{x,f})^2}{2\Sigma_x^2} + \frac{(\mu_y - \mu_{y,f})^2}{2\Sigma_y^2}]$$

Where:
N_c, N_f = number of cluster and field stars
$\sigma_x,\ \sigma_y$ = cluster proper motion dispersions
$\mu_{x,c}\ \mu_{y,c}$ = cluster proper motion zero point
$\Sigma_x,\ \Sigma_y$ = field proper motion dispersions
$\mu_{x,f}\ \mu_{y,f}$ = field proper motion zero point
Thus there are a total of 9 unknown parameters that need to be determined (since the total number of stars, N, is known, and $N = N_f + N_c$) The problem is given the proper motions for N stars to find the best fit for the nine parameters. The method of maximum likelihood, as discussed by Sanders (1971), is most commonly used.

There are two major problems with the method outlined above. One, the distribution of field stars in the PMVPD is a combination of the field stars space motions convolved with their distances, solar motion, and measuring errors, and is not close to being a normal distribution. Second, the parameters are intrinsically magnitude dependent. The first problem can be addressed by using different distribution functions for the field stars, and only fitting the field star distribution in the vicinity of the cluster. Depending on the offset of the center of the cluster distribution from the center of the field star distribution, investigators have used a normal distribution in one coordinate and an exponential in the other (Jones and Stauffer 1991) or a linear distribution. To account for the dependence of the parameters on magnitude, one can solve for the parameters in magnitude groupings, as was done by Francic (1989).

We note that *the membership probability is not a property of a star.* A star has a mass, radius, and effective temperature, but it does not have a membership probability. A star is either a cluster member or not. The membership probability, as discussed above, will depend on several factors. The errors of measurement will play a large role in the maximum membership probabilities, since larger errors will lead to smaller probabilities for cluster members. The limiting magnitude of the survey will also strongly affect the membership probabilities, since in faint surveys there will be a higher ratio of field stars to clusters stars and hence lower membership probabilities. Similarly, the size of the region surveyed will affect the membership probabilities, since in surveys over wide regions, there will be a higher ratio of field stars to clusters stars and hence lower membership probabilities. For these reasons, there can be no expectation that two investigations of the same region will yield identical membership probabilities. Moreover, two investigators using the same proper motion data set will not arrive at the same membership probabilities, because the probabilities will depend on how they they model the distributions and handle effects due to magnitude.

The meaning of membership probabilities is often misunderstood. If one solves for probabilities as outlined above, then the meaning is straightforward. If one picks n stars at random from the data set, then the number of stars that are members is the sum of the membership probabilities of the n stars. The misunderstanding that often occurs is that other criteria are used to pick a sample of stars, and in that case the number of member stars is not necessarily the sum of the membership probabilities. For example, if one chooses all stars in the data set that fall below the cluster main sequence, the number of cluster stars falling below the main sequence is not the sum of the membership probabilities of those stars. In fact, the number of such stars is most likely zero.

Another example of how proper motion membership probabilities can mislead is their use in determining the spatial structure of clusters. One method is to sum membership probabilities in rings around the clusters center. This will tend to make the cluster seem broader than it is, since the contribution of field stars with non-zero membership probabilities will have a constant density. Since the ratio of field stars to cluster stars increases to fainter magnitudes, the cluster will also appear to have larger extent at fainter magnitudes.

4. Use of Other Criteria

Vasilevskis (1962) strongly warned against the use of other criteria, such as color, magnitude, or position, in determining membership probabilities. He argued that the use of such criteria would invalidate the study for investigations such as the morphology of the color-magnitude diagram and the spatial structure of the cluster. As noted above, however, even membership studies using the classical Vasilevskis approach need to be used with care and sophistication for such studies, and in practice this is seldom the case. Moreover, at least in the case of the morphology of cluster color-magnitude diagrams, there has been little evidence of strange morphologies, such as stars significantly below the main sequence.

The search for faint cluster members has necessitated the use of criteria other than proper motions. Clusters are centrally concentrated, and for two stars with same proper motion, the one nearer the cluster center has a higher probability of being a cluster member. One can incorporate this directly into the determination of membership probabilities by assuming a model for the areal distribution of cluster stars (ρ_c) and field stars (ρ_f). Most often it is assumed that that the areal distribution for cluster stars depends only on the projected distance, r, from the cluster center and the distribution of field stars is constant. In this case,

$$P(\mu_x, \mu_y) = \frac{\rho_c(r)\Phi_c(\mu_x, \mu_y)}{\rho_f \Phi_f(\mu_x, \mu_y) + \rho_c(r)\Phi_c(\mu_x, \mu_y)}$$

van den Bergh and Sher (1960) give some empirical justification that clusters can be modeled with a distribution $\rho_c(r) = \rho_o e^{-r/r_o}$, where ρ_o and r_o are constants, and this is the model used by most investigators. Francic (1989), Jones and Stauffer (1991), and Hambly *et al.* (1995) discuss how to simultaneously solve for the constants describing the areal distribution and the proper motion distribution.

In proper motion surveys for very faint cluster members, the number of field stars so vastly exceeds the number of cluster stars that maximum membership probabilities approach zero, i.e., one can not determine membership. In such cases, one must use the color magnitude diagram as an additional criterion to have any hope of successfully finding cluster stars. The method used by several investigators is to find color and magnitudes for all stars in the vicinity of a cluster and construct a color magnitude diagram. Stars are chosen for proper motion measurement by their position in this diagram, where the expected position for cluster stars is determined either from the position of known cluster members or by the expected position determined from the the distance and reddening of the cluster.

5. Recent Advances

Over the last 20 years, computers and fast measuring machines have revolutionized the determination of cluster membership. The increasing power of computers has allowed more complicated modeling and this in turn has led to more extensive use of telescopes with optical distortions, such as Schmidts and reflectors with prime focus correctors, and to the use of different telescopes at different epochs. Fast measuring machines have allowed the measurement of proper motions of hundreds of thousands of faint stars in the vicinity of clusters.

Until recently, nearly all the cluster proper motion work was done with long focus refractors. This was partly because of the existence of first epoch plates taken with these telescopes, but partly also because the stability and lack of optical distortion led to simple transformations between two epochs. Computers have allowed more complicated models to be investigated. For example, when the Shane 3 m was put into operation at Lick Observatory in 1959, many plates were taken of open clusters at prime focus through a corrector. The corrector introduces a radial distortion that goes as odd powers of the distance from the optical center.

$$\Delta \mathbf{r} = \alpha \mathrm{r}^2 \mathbf{r} + \beta \mathrm{r}^4 \mathbf{r} + \ldots$$

where r is the distance from the distortion center. If the distortion center has (initially) unknown coordinates in the measuring system of $(\mathrm{x}_\circ, \mathrm{y}_\circ)$, than

$$\mathrm{r} = ((\mathrm{x} - \mathrm{x}_\circ)^2 + (\mathrm{y} - \mathrm{y}_\circ)^2)^{1/2}$$

One needs to determine not only the distortion constants α and β, but also the center of distortion, $\mathrm{x}_\circ$, $\mathrm{y}_\circ$, and this leads to equations of condition that are non-linear. Jones and Walker (1988) in their investigation of members of the Trapezium cluster discuss how these parameters can be determined using an iterative approach, but the solution would be too time consuming without computers.

Automatic measuring of images on photographic plates seriously began with installation of an automatic measuring machine at Lick Observatory in 1968 (Vasilevskis and Popov 1971). Although modernized several times since its installation and still it use, it has been superseded, at least in speed, by many other machines. For cluster work, the advent of very high speed measuring machines, coupled with computers for handling large data sets, has meant massive proper motion surveys to deep limits in the regions of the nearby clusters such as such as the Hyades, the Pleiades, Praesepe (NGC 2632), and α Persei.

As an example of the impact that high speed measuring and computing has had on cluster membership determinations, we consider Praesepe. The classic study of Klein-Wassink (1927) covered an area of about 3 square degrees to photographic magnitude 14. Six hundred stars were measured, and about 200 members found. It was nearly 40 years before a significant advance was made. Artyukhina (1966a, 1966b, 1971) identified probable cluster members in a cluster halo to $m_{pg} \sim 13$ and in the central region to $m_{pg} \sim 16$. Jones and Stauffer (1991) measured a color selected sample of 2,250 stars in a 16 square degree region to V=18. More recently, Hambly *et al.* (1995) have measured stars over a 19 square degree region on two first epoch glass copies of the Palomar Sky Survey and four plates taken with the United Kingdom Schmidt Telescope. The epoch difference was 38 years. Up to 150,000 stars were measured on each plates, and 57,000 stars, to a limiting R magnitude of near 20, survived into the final data base. For the membership analysis, they first selected stars that fell near the Praesepe main sequence, and then did a proper motion membership analysis on those stars. They found 515 highly probable members, 60% of which were new. The Hambly *et al.* study demonstrates the huge gains made over the last a few years. They not only used two different Schmidt telescopes, but the telescopes were in different hemispheres and the filters and emulsions were different. In spite of these problems, they significantly extended our knowledge of the membership of Praesepe stars.

6. Recent Membership Determinations

When Vasilevskis (1962) did a review of cluster proper motion work, he found 90 studies for 50 clusters. In the next 22 years, van Leeuwen (1984) reported on an additional 90 studies, but only 30 new clusters were added. For the present paper, I found an additional 60 studies, but only 10 new clusters were added. Most of the new studies aimed at extending existing studies either in area covered, limiting magnitude, or accuracy. In particular, much attention has been paid to finding very faint members of nearby clusters. Table I gives references to proper motion membership studies published since van Leeuwen's (1984) compilation. In Table I, the first column is the cluster, the second the size of the area covered in arc minutes, unless *sq deg* is used, in which case the area is in square degrees. The 3rd column is the limiting magnitude of the survey, and the fourth the reference, given more fully in the references.

7. Future Prospects

After a hundred years, the era of photographic astrometry is nearing an end. CCD's with their high quantum efficiency and linear response have already replaced the photographic plate in nearly all applications, including astrometry. CCD direct imaging gives us the capability to go much fainter and to obtain much higher precision. We can look forward to membership studies to faint magnitudes even for distant clusters, and because of the increased precision we will not have to wait 50 years for second epoch images. As we approach precisions of 1 milli-arcsecond in ground based astrometry, we have the prospect of doing meaningful work on distant clusters with epoch differences of 5-10 years. However, if we do not take and preserve the first epoch images now, they will not be available for future studies.

In addition, we should not forget the invaluable resource we have in the form of first epoch cluster plates. It is problematical if we will have the means of measuring these plates after the existing measuring machines are retired. The measurement of these plates, or some important sub-set, should be of high priority.

Tab. 1 Recent Membership Determinations

Cluster	Size	Mag Lim	Ref
NGC 188	60x50	17.5 B	Dinescu *et al.* 1996
NGC 225	30x30	16.1 B	Lattanzi *et al.* 1991
NGC 752	60x45	15 V	Francic 1989
NGC 752	110 *sq deg*	15 m_{pg}	Platais 1991a
NGC 1039	55x40	14 V	Ianna & Schlemmer 1993
NGC 1029	34 *sq deg*	16.2 V	Jones & Prosser 1996
NGC 1342	60x45	14.6 V	Francic 1989
NGC 1528	45x30	14.5 V	Francic 1989
NGC 1647	32x29	13.8 B	Geffert *et al.* 1996
NGC 1976	96x108	14 m_{pg}	Tian *et al.* 1996
NGC 1976	7 *sq deg*	14 B	McNamara *et al.* 1989
NGC 1976	60x40	12.5 V	van Altena *et al.* 1988
NGC 1976	24x332	16 I	Jones & Walker 1988
NGC 2168	28 *sq deg*	14.7 V	McNamara & Sekiguchi 1986
NGC 2281	60x45	16 V	Francic 1989
NGC 2286	120x90	14 V	Tian *et al* 1994
NGC 2287	55x40	12 V	Ianna *et al* 1987
NGC 2301	20x20	15 V	Marie 1992
NGC 2324	20x20	14.8 V	Marie 1993
NGC 2632	90x90	15.5 B	Wang *et al.* 1995
NGC 2632	19 *sq deg*	20 R	Hambly *et al.* 1995
NGC 2632	240x240	18 V	Jones & Stauffer 1991
NGC 2682	42x34	15.4 V	Girard *et al.* 1989
NGC 2682	90x90	15 B	Zhao *et al.* 1993
NGC 2682	80x80	15.6 V	Francic 1989
NGC 3680	60x40	18 B	Kozhurina-Platais *et al* 1995
NGC 6633	50x40	13 V	Francic 1989
NGC 6705			Shu *et al.* 1997
NGC 6939	18 *sq deg*	14.6 m_{pg}	Glushkova & Rastorgouev 1991
NGC 7092	110 *sq deg*	16.5	Platais 1984
NGC 7092	120x120		Glushkova 1991a
NGC 7209	55 *sq deg*	15.1 m_{pg}	Platais 1991b

Tab. 1 (cont.)

Cluster	Size	Mag Lim	Ref
IC 1396	150x150	14.5 m_{pg}	Glushkova 1991b
IC 1396	90x90	15 V	Marschall & van Altena 1987
IC 4665	40x30	15 V	Prosser 1993
IC 4725	45x30	14.5 V	Francic 1989
Coma	360x360	12	Bounatiro 1993
Hyades	112 *sq deg*	19 V	Reid1992, 1993
Hyades	300x300	20 E	Bryja *et al.* 1992, 1994
Pleiades	300x300	20 m_R	Hambly *et al.* 1991, 1993
Pleiades	200x200	19 B	Schilbach *et al.* 1995
Pleiades	90x90	14 m_{pg}	Shilbach 1991
Pleiades	240x240	18 V	Stauffer *et al.* 1991
Trumpler 10	65x65	12.8 m_v	Stock 1984
α Per	360x360	18.5 V	Prosser 1992
α Per	70x70	16.5 V	Stauffer *et al.* 1985
α Per	180x180	16.5 V	Stauffer *et al.* 1989

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